A new method for solving fully fuzzy linear programming problems by using the lexicography method

Mohammad Mehdi Shamooshaki[1], Ali Hosseinzadeh[1], Seyyed Ahmad Edalatpanah[2, *]

[1]Department of Mathematics, Comprehensive Imam Hossein University, Tehran, Iran
[2]Department of Applied Mathematics, Tonekabon Branch, Islamic Azad University, Tonekabon, Iran

Email address:

saedalatpanah@gmail.com (S. A. Edalatpanah)

Abstract: In this paper by using the lexicography method, we proposed a new model to solve fully fuzzy linear programming problem with L-R fuzzy number and find the fuzzy optimal solution of it. Our method has graceful structure and is easy to implement compared with some existing methods. To illustrate our method, a numerical example is solved.

Keywords: Fully Fuzzy Linear Programming, LR-Type Fuzzy Numbers, Ranking Function, Lexicography

1. Introduction

Linear Programming is that branch of mathematical programming which is designed to solve optimization problems where all the constraints as well as the objectives are expressed as Linear function. Linear programming is a method for finding the fittest answer from a range of possible answers. Linear programming is in two forms: classical linear programming and fuzzy linear programming in which the variables are assessed in a fuzzy manner.

In conventional approaches, the value of parameters in linear programming (LP) models must be well defined and precise. However, in real world environment, this is not a realistic assumption and there may exists uncertainty about the parameters. Hence, the parameters of linear programming problems may be represented as fuzzy numbers. The fuzzy LP problems (FLP) in which all the parameters as well as the variables are represented by fuzzy numbers are known as fully fuzzy linear programming (FFLP) problems.

The concept of fuzzy mathematical programming on general level was first proposed by Tanaka et.al [1]. The basic concepts of fuzzy decision making were first proposed by Bellman and Zadeh [2]. The first formulation of fuzzy linear programming (FLP) problems was proposed by Zimmermann [3]. Maleki et.al [4] used the concept of comparison of fuzzy numbers and proposed a new method for solving linear programming problems with fuzzy variables using an auxiliary problem. After wards, Lotfi et.al [5] presented a fully

fuzzy linear programming using lexicography method and fuzzy approximate solution. Moreover, Mishmast Nehi et.al [6] used the lexicographic ranking function to solve fuzzy number linear programming problems. Nasseri et.al [7, 8] developed the fuzzy primal simplex algorithms for solving both fuzzy number linear programming and FLP (Fuzzy Linear Programming) with fuzzy variables problems. Kumar et.al [9], proposed a new method for solving FFLP problems. However this method is limited to triangular fuzzy numbers and needs some corrections which is made by SaberiNajafi and Edalatpanah [10].

In this paper, we propose a new method for solving FFLPs with parameters in L-R fuzzy numbers by using lexicography method which generalizes [9]-[10]. We considered LR-type fuzzy numbers in this method which covers a wide class of fuzzy numbers, such as triangular fuzzy numbers, so our method gives the exact, not the approximate, feasible region of FFLPs, in comparison with [5].

2. Preliminaries

In this section some basic definitions, arithmetic operations and ranking function are reviewed. Below, we give definitions and notations taken from ([1], [3], [9-11]).

Definition 2.1. A function, usually denoted by L or R, is reference function of a fuzzy number iff $L(x)=L(-x)$, $L(0)=1$ and L is non-increasing on $[0,+\infty)$.

Definition 2.2. A fuzzy number $\tilde{M}$ is of LR-type, if there

exist reference functions L(for left), R(for right) and scalars $\alpha > 0, \beta > 0$ with

$$\mu_{\tilde{M}}(x) = \begin{cases} L\left(\dfrac{m-x}{\alpha}\right), & x \leq m \\ R\left(\dfrac{x-m}{\beta}\right), & x \geq m \end{cases}$$

The number (m) is called value of $\tilde{M}$, which is a real number and α and β are called the left and right spreads, respectively. We denoted $\tilde{M}$ by $(m, \alpha, \beta)_{LR}$.

Definition 2.3. Two L-R type fuzzy numbers $\tilde{M} = (m, \alpha, \beta)$ LR and $N = (n, \gamma, \delta)$ LR are said to be equal iff $m=n$ and $\alpha = \gamma$ and $\beta = \delta$.

Theorem 2.4. Let $\tilde{M} = (m, \alpha, \beta)_{LR}$ and $\tilde{N} = (n, \gamma, \delta)_{LR}$ be two fuzzy numbers of LR-type. Then we have:

1. $(m, \alpha, \beta)_{LR} \oplus (n, \gamma, \delta)_{LR} = (m + n, \alpha + \gamma, \beta + \delta)_{LR}$,

2. $-(m, \alpha, \beta)_{LR} = (-m, \beta, \alpha)_{RL}$,

3. $(m, \alpha, \beta)_{LR} \ominus (n, \gamma, \delta)_{LR} = (m - n, \alpha + \delta, \beta + \gamma)_{LR}$.

Definition 2.5. A L-R fuzzy number $(m, \alpha, \beta)_{LR}$ is said to be non-negative fuzzy number iff $m - \alpha \geq 0$.

Definition 2.6. A L-R fuzzy number $(m, \alpha, \beta)_{LR}$ is said to be non-negative fuzzy number iff $m - \alpha \leq 0$.

Theorem 2.7. Under the assumptions of Theorem 2.4,

1. For $\tilde{M}, \tilde{N}$ positive,

$(m, \alpha, \beta)_{LR} \otimes (n, \gamma, \delta)_{LR} \approx (mn, m\gamma + n\alpha, m\delta + n\beta)_{LR}$

2. For $\tilde{N}$ positive, $\tilde{M}$ negative,

$(m, \alpha, \beta)_{RL} \otimes (n, \gamma, \delta)_{LR} \approx (mn, n\alpha - m\delta, n\beta - m\gamma)_{RL}$

3. For $\tilde{M}, \tilde{N}$ negative

$(m, \alpha, \beta)_{LR} \otimes (n, \gamma, \delta)_{LR} \approx (mn, -n\beta - m\delta, -n\alpha - m\gamma)_{RL}$

Definition 2.8. A ranking function is a function $\Re: F(R) \to R$, where F(R) is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into a real line, where a natural order exists. Let $\tilde{M} = (m, \alpha, \beta)_{LR}$ be a LR-type fuzzy number, then

$\Re(\tilde{M}) = m + \dfrac{\beta - \alpha}{4}$ which, is equal with $\Re(\tilde{M}) = \int_{-\infty}^{+\infty} x\mu_{\tilde{M}}(x)dx / \int_{-\infty}^{+\infty} \mu_{\tilde{M}}(x)dx$.

Definition 2.9. Let $\tilde{M} = (m_1, \alpha_1, \beta_1)_{LR}$ and $\tilde{N} = (m_2, \alpha_2, \beta_2)_{LR}$ be two arbitrary LR-type fuzzy numbers. by using [11] we say that $\tilde{M}$ is relatively less than $\tilde{N}$, which is denoted by $\tilde{M} < \tilde{N}$, iff :

$m_1 < m_2$ or

$m_1 = m_2$ and $(\alpha_1 + \beta_1) > (\alpha_2 + \beta_2)$ or

$m_1 = m_2$, $(\alpha_1 + \beta_1) = (\alpha_2 + \beta_2)$ and $m_1 - \alpha_1 + \beta_1) < (2m2 - \alpha_2 + \beta_2)$

Remark 2.1. By using (*Definition 2.9*) and [11] we have:

$m_1 = m_2$, $(\alpha_1 + \beta_1) = (\alpha_2 + \beta_2)$,

and,

$(2m_1 - \alpha_1 + \beta_1) = (2m_2 - \alpha_2 + \beta_2)$ iff $\tilde{M} = \tilde{N}$.

3. FFLP Problem Formulation and Proposed Method

FFLP problems with m fuzzy equality constraints and n fuzzy variables may be formulated as follows:
Max (or Min) $\tilde{C}^T \otimes \tilde{X}$,

$$\text{s.t } \tilde{M} \otimes \tilde{X} = \tilde{b}, \qquad (3.1)$$

$\tilde{X}$ is non $-$ negetive fuzzy number.

W know that, $\tilde{C}^T = [\tilde{C}_j]_{1*n}$, $\tilde{X} = [\tilde{X}_j]_{n*1}$, $\tilde{M} = [\tilde{m}_{ij}]_{m*n}$, $\tilde{b} = [\tilde{b}_i]_{m*1}$ and $\tilde{C}_j$, $\tilde{X}_j$, $\tilde{m}_{ij}$, $\tilde{b}_i \in F(R)$ are all LR-type fuzzy numbers. Thus, FFLP (3.1) can be written as:
Max (or Min) $\sum_{j=1}^{n}(c_j, \xi_j, \zeta_j)_{LR} \otimes (x_j, \alpha_j, \beta_j)_{LR}$.
s.t.

$$\sum_{j=1}^{n}(n_{i,j}, \delta_{i,j}, \gamma_{i,j})_{LR} = (b_i, \rho_i, \eta_i)_{LR} \forall i=1,2,\ldots,m, \qquad (3.2)$$

$(x_j, \alpha_j, \beta_j)_{LR}$ is a non_negetive fuzzy number,

where

$\tilde{c}_j = (c_j, \xi_j, \zeta_j)_{LR}, \tilde{x}_j=(x_j, \alpha_j, \beta_j)_{LR}, \tilde{b}_i = (b_i, \rho_i, \eta_i)_{LR}, \tilde{m}_{i,j} = (m_{ij}, \mu_{ij}, \upsilon_{ij})_{LR}$

and $(m_{ij}, \mu_{ij}, \upsilon_{ij})_{LR} \otimes (x_j, \alpha_j, \beta_j)_{LR} = (n_{i,j}, \delta_{i,j}, \gamma_{i,j})_{LR}$.
We assume that all coefficient fuzzy numbers are positive. The similar procedure can be performed for the other cases.
Now, the steps of the proposed method are as follows:
Step 1. By Definition (2.8) and Remark (2.1) convert the FFLP problem (3.2) into the following linear programming :
Max (or Min) $\Re (\sum_{j=1}^{n}(c_j, \xi_j, \zeta_j)_{LR} \otimes (x_j, \alpha_j, \beta_j)_{LR}$.
s. t.

$$\sum_{j=1}^{n}(n_{ij}) = b_i \qquad \forall i=1,2,\ldots,m,$$

$$\sum_{j=1}^{n}(\delta_{ij} + \gamma_{ij}) = (\rho_i + \eta_i) \quad \forall i=1,2,\ldots,m, \qquad (3.3)$$

$$\sum_{j=1}^{n}(2n_{ij} - \delta_{ij} + \gamma_{ij}) = (2 b_i - \rho_i + \eta_i) \quad \forall i=1,2,\ldots,m$$

$$x_j, \alpha_j, \beta_j \geq 0 \qquad \forall j=1,2,\ldots,n$$

Step 2. Find the optimal solution of (3.3), say x^*_j, α^*_j and β^*_j.

Step 3. Find the fuzzy optimal solution of FFLP problem (3.2), by putting $x^*_j=\left(x^*_j, \alpha^*_j, \beta^*_j\right)_{LR}$.

Step 4. Find the fuzzy optimal objective value by putting x^*_j in:

$$\sum_{j=1}^{n}(c_j, \xi_j, \zeta_j)_{LR} \otimes \left(x^*_j, \alpha^*_j, \beta^*_j\right)_{LR}.$$

In continuance, we have solved an example using of lexicography method and Mathematica 9 software, which was

already solved by Kumar et al[9].

Example 3.1. Let us consider the following FFLP and solve it by the proposed method

Max (or Min) $(((2,1,1)_{LR})\,\tilde{x}_1 \oplus ((3,1,1)_{LR})\tilde{x}_2)$

s.t.

$$((1,1,1)_{LR})\tilde{x}_1 \oplus ((2,1,1)_{LR})\,\tilde{x}_2 = (10,8,14)_{LR}$$

$$((2,1,1)_{LR})\,\tilde{x}_1 \oplus ((1,1,1)_{LR})\,\tilde{x}_2 = (8,7,13)_{LR}$$

$\tilde{x}_1 \ and \ \tilde{x}_2$ are non-negative LR-type fuzzy numbers,

In view of (3.3) and definition (2.9) and remark (2.1), we can write this FFLP as:

Max 2 $x_1 + 3x_2 + \frac{1}{4}(2(\beta_1 - \alpha_1) + 3(\beta_2 - \alpha_2)$

s.t.

$x_1 + 2x_2 = 10$

$2x_1 + 2x_2 + \beta_1 + 2\beta_2 + \alpha_1 + 2\alpha_2 = 22$

$2x_1 + 4x_2 - \alpha_1 - 2\alpha_2 + \beta_1 + 2\beta_2 = 26$

$2x_1 + x_2 = 8$

$2x_1 + 2x_2 + 2\alpha_1 + \alpha_2 + 2\beta_1 + \beta_2 = 20$

$4x_1 + 2x_2 - 2\alpha_1 - \alpha_2 + 2\beta_1 + \beta_2 = 22$

The optimal solution of the LP problem, is $x_1 = 2$, $\alpha_1 = 0$, $\beta_1 = 2$, $x_2 = 4$, $\alpha_2 = 1$, $\beta_2 = 3$.

Then the optimal solution of the FFLP problem, obtained in Step 3, is $\tilde{x}_1 = (2,0,2)_{LR}$ and $\tilde{x}_2 = (4,1,3)_{LR}$. Step 4 yields the optimal fuzzy objective value to be $(16,9,\frac{37}{2})_{LR}$ .This problem is also solved by [9], for which the authors implemented the triangular fuzzy numbers. However, our method permits a great choice of auxiliary reference functions L and R, which in a special case; it covers the triangular fuzzy numbers. Also, our proposed method is easy to implement compared to [5].

4. Conclusions

In this paper, a new method is represented to solve FFLP problems. This method based on a new lexicography on L-R fuzzy numbers. Here we witness an example (problem) solved by using lexicographic method which was previously solved by using rank function. We observed, our method is far more efficient.

References

[1] H. Tanaka, T. Okuda and K. Asai, " On fuzzy mathematical programming", J.Cybernetics,3 (1974) 37–46.

[2] R.E. Bellman and L.A. Zadeh, " Decision making in a fuzzy environment", Management Sci, 17 (1970) 141–164.

[3] H.J. Zimmermann, " Fuzzy programming and linear programming with several objective functions", Fuzzy Sets and Systems, 1 (1978) 45–55.

[4] H.R. Maleki, M. Tata and M. Mashinchi, " Linear Programming with fuzzy variables ", Fuzzy Sets and Systems, 109 (2002) 21-33.

[5] F.H. Lotfi, T. Allahviranloo, M.A. Jondabeha and L. Alizadeh, " Solving a fully fuzzylinear programming using lexicography method and fuzzy approximate solution", Appl. Math. Modell, 33 (2009) 3151-3156.

[6] H. MishmastNehi, H.R. Maleki, and M. Mashinchi, "Solving fuzzy number linear programming problem by lexicographic ranking function", Italian Journal of Pure and Applied Mathematics, 15 (2004) 9-20.

[7] S.H. Nasseri and E. Ardil, "Simplex Method for Fuzzy Variables Linear Programming Problems", World Academy of Science-Engineering and Technology, 8 (2005) 198-202.

[8] S.H. Nasseri , E. Ardil , A. Yazdani and R. Zaefarian, "Simplex Method for Solving Linear Programming Problems with Fuzzy Numbers", World Academy of Science-Engineering and Technology, 10 (2005) 284-288.

[9] A. Kumar, J. Kaur and P. Singh, "A new method for solving fully fuzzy linear programming problems", Appl. Math. Modell, 35 (2011) 817-823.

[10] Saberi Najafi, H., and S. A. Edalatpanah, "A Note on A new method for solving fully fuzzy linear programming problems", Appl. Math. Modell,37 (2013)7865–7867.

[11] R. Ezzati,E. Khorram,R. Enayati," A New Algorithm to Solve Fully Fuzzy Linear Programming Problems Using the MOLP Problem ", Appl. Math. Modelling, (2013) http://dx.doi.org/10.1016/j.apm.2013.03.014.