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# ON (m, n) –upper Q-fuzzy soft subgroups

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**Abstract:** In this paper we shall study some properties for upper Q- fuzzy subgroups, some lemma and theorem for this subject. We shall study the upper Q- fuzzy index with the upper fuzzy sub groups; also we shall give some new definitions for this subject. On the other hand we shall give the definition of the upper normal fuzzy subgroups, and study the main theorem for this. We shall also give new results on this subject.

**Keywords:** Fuzzy Set, Soft Set, Fuzzy Soft Set, (m, n) –Upper Q-Fuzzy Soft Group, Product, Upper Q-Fuzzy Order, Upper Q-Fuzzy Cossets, Upper Q-Fuzzy Index

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## 1. Introduction

Fuzzy sets was first introduced by Zadeh [20 ] and then the fuzzy sets have been used in the reconsideration of classical mathematics. Yuan et al. [17] introduced the concept of fuzzy subgroup with thresholds. A fuzzy subgroup with thresholds  $\lambda$  and  $\mu$  is also called a  $(\lambda, \mu)$ -fuzzy subgroup. Yao continued to research  $(\lambda, \mu)$ -fuzzy normal subgroups,  $(\lambda, \mu)$ -fuzzy quotient subgroups and  $(\lambda, \mu)$ -fuzzy sub rings in [18,19 ]. Shen researched anti-fuzzy subgroups in [11] and Dong [6] studied the product of anti-fuzzy subgroups. Molodtsov [7] introduced the concept of soft sets that can be seen as a new mathematical theory for dealing with uncertainty. Molodtsov applied this theory to several directions [7], and then formulated the notion of soft number, soft derivative, soft integral, etc in [8] , the soft set theory has been applied to many different fields with great success. Maji et.al [8] worked on theoretical study of soft sets in detail presented an application of soft set in the decision making problem using the reduction of soft sets.

Majiet.al [9] presented the concept of fuzzy soft sets by embedding the ideas of fuzzy sets. By using this definition of fuzzy soft sets many interesting applications of soft set theory have been expanded by some Researchers. Roy and Maji [9] gave some applications of fuzzy soft sets. A.Solairaju and R.Nagarajan introduced the concept of Structures of Q- fuzzy groups [12]. A.Solairaju and R.Nagarajan studied some structure properties of upper Q- fuzzy index order with upper Q- fuzzy subgroups[13].In this

paper, We analyze the notions of (m, n)-Upper Q- fuzzy subgroups, studied some properties for upper Q- fuzzy subgroups, some propositions and theorem for this subject and discussed the product of them.

## 2. Preliminaries

This section, briefly reviews the basic characteristics of fuzzy set and fuzzy soft sets.

*Definition 2.1:* [19] By a fuzzy subset of a nonempty set X we mean a mapping from X to the unit interval [0,1].

*Definition 2.2:* [11] If A is a fuzzy subset of X, then we denote  $A(\alpha) = \{x \in X | A(x) \leq \alpha\}$  for all  $\alpha \in [0,1]$ .

Throughout this article, we will always assume that

$$0 \leq m < n \leq 1.$$

Let  $G, G_1,$  and  $G_2$  always denote groups in the following.  $I, I_1,$  and  $I_2$  are identities of  $G, G_1,$  and  $G_2,$  respectively.

*Definition 2.3:*[7] Let U be an initial universe, P (U) be the power set of U, E be the set of all parameters and  $A \subseteq E$ . A soft set  $(f_A, E)$  on the universe U is defined by the set of order pairs

$$(f_A, E) = \{(e, f_A(e)) : e \in E, f_A \in P(U)\} \text{ where } f_A : E \rightarrow P(U) \text{ such that } f_A(e) = \phi \text{ if } e \notin A.$$

Here  $f_A$  is called an approximate function of the soft set.

*Example:* Let  $U = \{u_1, u_2, u_3, u_4\}$  be a set of four shirts and  $E = \{\text{white}(e_1), \text{red}(e_2), \text{blue}(e_3)\}$  be a set of parameters. If  $A = \{e_1, e_2\} \subseteq E$ ,  $f_A(e_1) = \{u_1, u_2, u_3, u_4\}$  and  $f_A(e_2) =$

$\{u_1, u_2, u_3\}$ , then we write the soft set  $(f_A, E) = \{(e_1, \{u_1, u_2, u_3, u_4\}), (e_2, \{u_1, u_2, u_3\})\}$  over  $U$  which describe the "colour of the shirts" which Mr. X is going to buy.

**Definition 2.4:**[9] Let  $U$  be an initial universe,  $E$  be the set of all parameters and  $A \subseteq E$ . A pair  $(F, A)$  is called a fuzzy soft set over  $U$  where  $F: A \rightarrow P(U)$  is a mapping from  $A$  into  $P(U)$ , where  $P(U)$  denotes the collection of all fuzzy subsets of  $U$ .

**Example:** Consider the above example, here we cannot express with only two real numbers 0 and 1, we can characterized it by a membership function instead of crisp number 0 and 1, which associate with each element a real number in the interval  $[0,1]$ . Then

$$(f_A, E) = \{f_A(e_1) = \{(u_1, 0.7), (u_2, 0.5), (u_3, 0.4), (u_4, 0.2)\},$$

$f_A(e_2) = \{(u_1, 0.5), (u_2, 0.1), (u_3, 0.5)\}\}$  is the fuzzy soft set representing the "colour of the shirts" which Mr. X is going to buy.

**Definition 2.5:** A fuzzy soft set  $A$  of a group  $G$  is called  $(m,n)$ - Upper Q-fuzzy soft subgroup of  $G$  if for all  $x, y, z \in G$ , [UFSG1]  $A(xy, q) \wedge m \leq \max\{A(x, q), A(y, q)\} \vee .n$  and [UFSG2]  $A(x^{-1}, q) \wedge m \leq A(x, q) \vee .n$  where  $x^{-1}$  is the inverse element of  $x$ .

Throughout this article,  $\max\{A(x, q), A(y, q)\}$  is sometimes replaced as  $A(x, q) \vee . A(y, q)$ .

**Proposition-1:** If  $A$  is  $(m,n)$  -upper Q-fuzzy soft group of  $G$ , then  $A(1, q) \wedge m \leq A(x, q) \vee .n$  for all  $x \in G$ , when  $1$  is the identity of  $G$ .

**Proof:** For all  $x \in G$  and let  $x^{-1}$  be the inverse element of  $x$ . Then  $A(1, q) \wedge m = A(xx^{-1}, q) \wedge m = (A(xx^{-1}, q) \wedge m) \wedge m \leq \max\{A(x, q), A(x^{-1}, q)\} \vee .n = \max\{A(x, q) \wedge m, A(x^{-1}, q) \wedge m\} \vee .n (n \vee .m) = A(x, q) \vee .n$ .

### 3. Main Results

**Theorem-3.1:** Let  $A$  be upper Q-fuzzy soft subset of a group  $G$ . Then  $A$  is  $(m,n)$ -upper Q-fuzzy soft group of  $G$  if and only if  $A(x^{-1}y, q) \wedge m \leq \max\{A(x, q), A(y, q)\} \vee .n$  for all  $x, y \in G$ .

**Proof:** Let  $A$  is  $(m,n)$ -upper Q-fuzzy soft group of  $G$ . Then  $A(x^{-1}y, q) \wedge m = A(x^{-1}y, q) \wedge m \wedge m \leq (\max\{A(x^{-1}, q), A(y, q)\} \vee .n) \wedge m = (A(x^{-1}, q) \wedge m, A(y, q)) \vee .(n \wedge m) \leq ((A(x, q) \vee .n) \vee (A(y, q))) \vee .n = \max\{A(x, q), A(y, q)\} \vee .n$ .

Conversely, suppose  $A(x^{-1}y, q) \wedge m \leq \max\{A(x, q), A(y, q)\} \vee .n$  for all  $x, y \in G$ , then  $A(1, q) \wedge m = A(x^{-1}x, q) \wedge m \leq \max\{A(x, q), A(y, q)\} \vee .n = A(x, q) \vee .n$ .

So

$$(A(x^{-1}, q) \wedge m) \wedge m = (A(x^{-1}.1, q) \wedge m) \wedge m = (A(x^{-1}, q) \wedge m) \wedge m \leq \max\{A(x^{-1}, q), A(y, q)\} \vee .n \wedge m$$

$$= (A(x^{-1}, q) \wedge m) \vee .(A(y, q) \vee .n) \wedge m \leq \max\{(A(x, q) \vee .n), (A(y, q)) \vee .n\}$$

$= \max\{A(x, q), A(y, q)\} \vee .n$ . So  $A$  is a  $(m,n)$ -upper Q-fuzzy soft group of  $G$ .

**Theorem-3.2:** Let  $A$  be a Q-fuzzy soft subset of a group. Then the following are equivalent

(i).  $A$  is  $(m,n)$ -upper Q-fuzzy soft subgroup of  $G$ . (ii)  $A(\alpha)$  is a subgroup of  $G$ , for any  $\alpha \in [m, n]$ , where  $A(\alpha) \neq \Phi$ .

**Proof:** "(i) implies (ii)"

Let  $A$  be  $[m, n]$ -upper Q-fuzzy soft subgroup of  $G$ . For any  $\alpha \in [m, n]$ , such that  $A(\alpha) \neq \Phi$ , we need to show that  $x^{-1}y \in A(\alpha)$ , for all  $x, y \in A(\alpha)$ . Since  $A(x, q) \leq \alpha$  and  $A(y, q) \leq \alpha$ , then  $(A(x^{-1}, q) \wedge m \leq \max\{A(x, q) \vee .n, (A(y, q)) \vee .n\} \leq \max\{\alpha, \alpha\} \vee .n = \alpha \vee .n = \alpha$ . Note that  $\alpha \leq n$ , we obtain  $(A(x^{-1}y, q) \leq \alpha$ . so  $x^{-1}y \in A(\alpha)$ .

"(ii) implies (i)"

Conversely, let  $A(\alpha)$  is a subgroup of  $G$ . we need to prove that  $(A(x_1, q) \wedge m) \leq \max\{A(x, q) \vee .n\}$ , for all  $x \in G$ . If there exist  $x_0, y_0 \in G$  such that  $A(x_0^{-1}y_0, q) \wedge m = \alpha \geq \max\{A(x_0, q), A(y_0, q)\} \vee .n$ , then  $A(x_0, q) \leq \alpha$ ,  $A(y_0, q) \leq \alpha$  and  $\alpha \in [m, n]$ . Thus  $x_0 \in A(\alpha)$ .

And  $y_0 \in A(\alpha)$ . But then  $A(x_0^{-1}y_0, q) \geq \alpha$ , that is  $x_0^{-1}y_0$  does not belong to  $A(\alpha)$ . This is a contradiction with that  $A(\alpha)$  is a subgroup of  $G$ . Hence  $A(x^{-1}y, q) \wedge m \leq \max\{(A(x, q) \vee .n), (A(y, q)) \vee .n\}$  holds for any  $x, y \in G$ . Therefore  $A$  is a  $(m,n)$  - upper Q-fuzzy soft subgroup of  $G$ . we get  $\Phi = 1$ , where  $\Phi$  is the empty set.

**Theorem-3.3:** Let  $f: G_1 \rightarrow G_2$  be a homomorphism and let  $A$  be  $(m,n)$ -upper Q-fuzzy soft subgroup of  $G_1$ . Then  $f(A)$  is  $(m,n)$ -upper Q-fuzzy soft subgroup of  $G_2$ , where  $f(A)(\delta, q) = \inf\{A(x, q) \mid f(x) = \delta\}$ , for all  $\delta \in G_2$  and  $x \in G$ .

**Proof:** If  $f^{-1}(y_1) = \Phi$  or  $f^{-1}(y_2) = \Phi$  for any  $y_1, y_2 \in G_2$ , then  $(f(A)(\delta_1^{-1}\delta_2, q) \wedge m) \leq 1 = \max\{f(A)(\delta_1, q), f(A)(\delta_2, q)\} \vee .n$ .

Suppose that  $f^{-1}(y_1) \neq \Phi$  or  $f^{-1}(y_2) \neq \Phi$  for any  $y_1, y_2 \in G_2$ . Then For any  $y_1, y_2 \in G_2$ , we have

$$(f(A)(\delta_1^{-1}\delta_2, q) \wedge m) = \inf\{A(t, q) \mid f(t) = \delta_1^{-1}\delta_2\} \wedge m \text{ as } t \in G.$$

$$= \inf\{A(t, q) \mid f(t) = \delta_1^{-1}\delta_2\} \wedge m \text{ as } t \in G_1 \leq \inf\{A(x_1^{-1}x_2, q) \wedge m \mid f(x_1) = \delta_1, f(x_2) = \delta_2\} \text{ as } x_1, x_2 \in G_1.$$

$$\leq \inf\{\max\{A(x, q), A(y, q)\} \vee .n \mid f(x_1) = \delta_1, f(x_2) = \delta_2\} \text{ as } x_1, x_2 \in G_1.$$

$$\leq \inf\{\max\{A(x_1, q) \mid f(x_1) = \delta_1\}, \inf\{\max\{A(x_2, q) \mid f(x_2) = \delta_2\}\}$$

$$= \max\{f(A)(\delta_1, q), f(A)(\delta_2, q)\} \vee .n$$

So  $f(A)$  is  $(m,n)$ -upper Q-fuzzy soft subgroup of  $G_2$ .

**Theorem-3.4:** Let  $f: G_1 \rightarrow G_2$  be a homomorphism and let  $B$  be  $(m,n)$ -upper Q-fuzzy soft subgroup of  $G_2$ . Then  $f^{-1}(B)$  is  $(m,n)$ - upper Q-fuzzy soft subgroup of  $G_1$ , where  $f^{-1}(B)(x, q) = B(f(x, q))$ , for all  $x \in G_1$ .

**Proof:** For any  $x_1, x_2 \in G_1$ .

$$f^{-1}(x_1^{-1}x_2, q) \wedge m = B(f(x_1^{-1}x_2, q)) \wedge m$$

$$= B((f(x_1, q))^{-1} f(x_2, q)) \wedge m \leq \max\{B(f(x_1, q), B(f(x_2, q)) \vee .n$$

$$= \max \{f^1(B)(x_1, q), f^1(B)(x_2, q)\} \vee .n$$

So,  $f^1(B)$  is (m,n)-upper Q-fuzzy subgroup of  $G_1$ .

Let  $G_1$  be a group with the identity  $I_1$  and  $G_2$  be a group with the identity  $I_2$ , then  $G_1 \times G_2$  is a group with the identity  $(I_1, I_2)$  if we define  $(x_1, y_1)(x_2, y_2) = (x_1x_2, y_1y_2)$  for all  $(x_1, y_1), (x_2, y_2) \in G_1 \times G_2$ . Moreover, the inverse element of any  $(x, a) \in G_1 \times G_2$  is  $(y, b) \in G_1 \times G_2$  if and only if  $y$  is the inverse element of  $x$  in  $G_1$  and  $b$  is the inverse element of  $a$  in  $G_2$ .

**Theorem-3.5:** Let  $A$  and  $B$  be two (m,n)-upper Q-fuzzy soft subgroups of groups  $G_1$  and  $G_2$  respectively. The Product of  $A$  and  $B$  denoted by  $A \times B$  is (m,n)- upper Q-fuzzy soft subgroup of the group  $G_1 \times G_2$ , where  $(A \times B)(x, y)q = \max \{A(x, q), A(y, q)\}$ , for all  $(x, y) \in G_1 \times G_2$ .

**Proof:** Let  $(x^{-1}, a^{-1})$  be the inverse element of  $(x, q)$  in  $G_1 \times G_2$ . Then  $x^{-1}$  is the inverse element of  $x$  in  $G_1$  and  $a^{-1}$  is the inverse element of  $a$  in  $G_2$ . Hence  $A(x^{-1}, q) \wedge m \leq A(x, q) \vee .n$  and  $B(a^{-1}, q) \wedge m \leq B(a, q) \vee .n$ .

For all  $(y, b) \in G_1 \times G_2$ , we have

$$\begin{aligned} & ((A \times B)((x, a)^{-1}(y, b)) \wedge m = ((A \times B)((x^{-1}, a^{-1})(y, b))q) \wedge m \\ & = \max \{A(x^{-1}y, q), B(a^{-1}b, q)\} \wedge m \\ & = \max \{A(x^{-1}y, q) \wedge m, B(a^{-1}b, q) \wedge m\} \\ & \leq \max \{\max (A(x, q), A(y, q)) \vee .n, \max (B(a, q), B(b, q)) \vee .n\} \\ & = \max \{\max (A(x, q), B(a, q)), \max (A(y, q), B(b, q))\} \vee .n \\ & = \max \{A \times B (x, a)q, A \times B (y, a)q\} \vee .n \end{aligned}$$

Hence  $A \times B$  is (m,n)-upper Q-fuzzy soft subgroup of  $G_1 \times G_2$ .

**Theorem-3.6:** Let  $A$  and  $B$  be two (m,n)-upper Q-fuzzy soft subsets of groups  $G_1$  and  $G_2$ , respectively. If  $A \times B$  is (m,n)- upper Q-fuzzy soft subgroup of the group  $G_1 \times G_2$ , then at least one of the following statements must hold.

- (i).  $A(I_1, q) \wedge m \leq B(a, q) \vee .n$ , for all  $a \in G_2$  and
- (ii).  $B(I_2, q) \wedge m \leq A(x, q) \vee .n$ , for all  $x \in G_1$ .

**Proof:** Let  $A \times B$  is (m,n)- upper Q-fuzzy soft subgroup of the group  $G_1 \times G_2$ .

By composition, suppose that none of the statements hold. Then we can find  $x \in G_1$  and  $a \in G_2$  such that  $A(x, q) \vee .n \leq B(I_2, q) \wedge m$  and  $B(a, q) \vee .n \leq A(I_1, q) \wedge m$ . Now

$$\begin{aligned} & ((A \times B)(x, a)q) \vee .n = \max \{A(x, q), B(y, q)\} \vee .n \\ & = \max \{A(x, q) \vee .n, B(y, q) \vee .n\} \\ & \leq \max \{A(I_1, q) \wedge m, B(I_2, q) \wedge m\} = (A \times B)(I_1, I_2)q \wedge m. \end{aligned}$$

Thus  $A \times B$  is (m,n)- upper Q-fuzzy soft subgroup of the group  $G_1 \times G_2$  satisfying  $((A \times B)(x, a)q) \vee .n \leq (A \times B)(I_1, I_2) \wedge m$ . This is a contradict with that  $(I_1, I_2)$  is the identity of  $G_1 \times G_2$ .

**Theorem-3.7:** Let  $A$  and  $B$  be Q-fuzzy subsets of groups  $G_1$  and  $G_2$  respectively, such that  $B(I_2, q) \wedge m \leq A(x, q) \vee .n$  for all  $a \in G_1$ . If  $A \times B$  is a (m,n)-upper Q-fuzzy soft subgroup of  $G_1 \times G_2$ , then  $A$  is a (m,n)-upper Q-fuzzy soft subgroup of  $G_1$ .

**Proof:** From  $B(I_2, q) \wedge m \leq A(x, q) \vee .n$  we obtain that  $m \leq$

$A(x, q) \vee .n$  or  $B(I_2, q) \wedge m \leq A(x, q) \vee .n$  for all  $a \in G_1$ .

Let  $x, y \in G_1$ . Then  $(x, I_2), (y, I_2) \in G_1 \times G_2$ .

Two cases are possible.

- (1) If  $m \leq A(x, q) \vee .n$  for all  $a \in G_1$ , then  $A(xy, q) \wedge m \leq m \leq A(x, q) \vee .n \leq \max \{A(x, q), A(y, q)\} \vee .n$  and  $A(I_1, q) \wedge m \leq m \leq A(x, q) \vee .n$ .
- (2) If  $B(I_2, q) \wedge m \leq A(x, q) \vee .n$  for all  $x \in G_1$ . Then

$$(A(xy, q) \wedge m) \leq (A(xy, q) \vee B(I_2, q)) \wedge m = (A \times B)(xy, I_2)q \wedge m$$

$$= (A \times B)((x, I_2)(y, I_2)q) \wedge m \leq \max \{(A \times B)((x, I_2)q), (A \times B)((y, I_2)q)\} \vee .n$$

$$\leq \max \{(A(x, I_2), B(I_2, q)), (A(y, I_2), B(I_2, q))\} \vee .n = \max \{A(x, q), A(y, q)\} \vee .n$$

$$\text{And } A(I, q) \wedge m \leq (A(I, q) \cup B(I_2, q)) \wedge m = ((A \times B)(I_1, I_2)q) \wedge m$$

$$\leq ((A \times B)(x, I_2)q) \vee .n = \max \{A(x, q), B(I_2, q)\} \vee .n = A(x, q) \vee .n$$

Hence  $A$  is (m, n)-upper Q-fuzzy soft subgroup of  $G_1$ .

Analogously, we have

**Theorem-3.8:** Let  $A$  and  $B$  be Q-fuzzy subsets of groups  $G_1$  and  $G_2$  respectively, such that  $A(I_1, q) \wedge m \leq B(a, q) \vee .n$  for all  $a \in G_2$ . If  $A \times B$  is (m, n)-upper Q-fuzzy soft subgroup of  $G_1 \times G_2$ , then  $B$  is a (m, n)-upper Q-fuzzy soft subgroup of  $G_2$ .

From the Previous theorem, we have the following corollary.

**Corollary 3.9:** Let  $A$  and  $B$  be Q-fuzzy subsets of groups  $G_1$  and  $G_2$  respectively, such that  $A(I_1, q) \wedge m \leq B(a, q) \vee .n$  for all  $a \in G_2$ . If  $A \times B$  is (m, n)-upper Q-fuzzy soft subgroup of  $G_1 \times G_2$ , then either  $A$  is (m, n)-upper Q-fuzzy soft subgroup of  $G_1$  or (m, n)-upper Q-fuzzy soft subgroup of  $G_2$ .

**Section-4 (m, n)-Upper Q- fuzzy index order with fuzzy soft subgroups**

**Definition 4.1:** Let  $\lambda$  be an (m,n)-upper Q-fuzzy soft subgroup of  $G$ . For any  $x \in G$ , the smallest positive integer  $n$  such that  $\lambda(x^n) \wedge m = \lambda(e) \vee .n$  is called (m,n)- upper Q-fuzzy soft order of  $x$ . If there does not exist such  $n$  then  $x$  is said to have an infinite upper Q- fuzzy soft order. We shall denote the upper Q-fuzzy soft order of  $x$  by  $O(\lambda(x))$ .

**Example:** Let  $G = \{e, a, b, ab\}$  be the Klein four group and let  $A = \{(e, 1/4), (a, 3/4), (b, 3/4), (ab, 1/4)\}$  be an (m,n)- upper Q-fuzzy soft subgroup, then  $O(A(ab, q)) = 1$  and  $O(A(a, q)) = 2$ .

**Definition 4.2:** A (m,n)- upper Q-fuzzy soft subgroup  $A_H$  of  $G$  is called (m,n)- upper Q- fuzzy soft normal subgroup of  $G$  if  $A_H(xy, q) \wedge m = A_H(yx, q) \vee .n$  for all  $x, y$  of  $G$  and  $q \in Q$ .

**Definition 4.3:** Let  $A$  be an (m,n)- upper Q-fuzzy soft subgroup of a group  $G$ . A Q-fuzzy soft subset satisfying:  $[gA](x, q) \wedge m = A(g^{-1}x, q) \vee .n$  for all  $x, g \in G$ ; is called an upper Q-fuzzy soft left cosets of  $A$ , while that satisfies:  $[Ag](x, q) \wedge m = A(xg^{-1}, q) \vee .n$  for all  $x, g \in G$ ; is called an upper Q-fuzzy soft right cosets of  $A$ .

In the following study, the upper Q-fuzzy soft left and right cosets must be fuzzy soft sets but not necessarily be

(m,n)- upper Q-fuzzy soft subgroups.

The following propositions are obtained from the above definitions

*Proposition 4.1:* Let A be an (m,n)- upper Q-fuzzy soft subgroup of the group G. Then for any integer n and  $x \in G$ , we have  $A(x^n, q) \wedge m \leq A(x, q) \vee n$

*Proposition 4.2:* Let G be a group and let A be an (m,n)-upper Q-fuzzy soft subgroup of the group G; let  $x \in G$  be of finite order k ; if  $r \in \mathbb{N}$  and k are relatively prime, then  $A(x^r, q) \wedge m = A(x, q) \vee n$ .

*Proof*

Since r, k is relatively prime, then by Bizout Theorem There exists a, b  $\in \mathbb{Z}$  such that  $1 = ar + bk$  ; therefore  $A(x, q) \wedge m = A(x^{ar+bk}, q) \wedge m = A((x^{ar} \cdot x^{bk}), q) \wedge m = A(x^{ar}, q) \wedge m \leq A(x^r, q) \vee n \leq A(x, q) \vee n$ . Then we get  $A(x, q) \wedge m \leq A(x^r, q) \wedge m \leq A(x, q) \vee n$  therefore  $A(x^r, q) \wedge m = A(x, q) \vee n$ .

*Lemma 4.3:* Let A be an (m,n)- upper Q- fuzzy soft subgroup of the group G, for  $x \in G$ . If  $A(e, q) \wedge m = A(x^n, q) \vee n$  for some  $n \in \mathbb{Z}$ , then  $O(A(x, q))$  divides n .

*Proof*

Suppose that  $O(A(x, q)) = k$  , then by Euclidean Algorithm , there exists a , b  $\in \mathbb{Z}$  . Such that  $n = ka + b$ ;  $0 \leq b < k$ ; thus  $A(x^b, q) \wedge m = A(x^{n-ka}, q) \wedge m = A(x^n (x^k)^{-a}, q) \wedge m \leq \max \{ A(x^n, q), A(x^k)^{-a}, q \} \vee n \leq \max \{ A(e, q), A(x^k, q) \} \vee n = \max \{ A(e, q), A(e, q) \} \vee n = A(e, q) \vee n$ . Thus  $A(x^b, q) \wedge m \leq A(e, q) \vee n$ ; also  $A(x^b, q) \wedge m \geq A(e, q) \vee n$ , then we get  $A(x^b, q) \wedge m = A(e, q) \vee n$ . Then  $b = 0$ ; also  $n = ka$  which is given  $O(A(x, q))$  divides n.

*Proposition 4.4:* Let A be an(m,n)- upper Q-fuzzy soft subgroup of the group G, let  $O(A(x, q)) = A$ ;  $x \in G$ . If k  $\in \mathbb{Z}$  such m, n relatively prime, then  $A(x^k, q) = A(x, q)$ .

*Proof*

Since m, k relatively prime,  $(m, k) = 1$  Then there exists a, b  $\in \mathbb{Z}$  such that  $ma + kb = 1$   $A(x, q) \wedge m = A(x^{ma+kb}, q) = A((x^m)^a \cdot (x^k)^b, q) \wedge m \leq \max \{ A(x^m, q)^a, A(x^k, q)^b \} \vee n \leq \max \{ A(x^m, q), A(x^k, q) \} \vee n \leq \max \{ A(e, q), A(x^k, q) \} \vee n = A(x^k, q) \vee n \leq A(x, q) \vee n$

Therefore  $A(x^k, q) \wedge m = A(x, q) \vee n$ .

*Lemma 4.5 :* Let A be an(m,n)- upper Q- normal fuzzy soft subgroup of the group G. Then  $O(A(x, q)) \wedge m = O(A(y^{-1}xy, q)) \vee n$  for all  $x, y \in G$ .

*Proof*

Let  $x, y \in G$ , then we have  $A(x^m) \wedge m = A(y^{-1}xy) \wedge m = A((y^{-1}xy)^m) \wedge m$ , for all  $m \in \mathbb{Z}$ . Thus  $O(A(x, q)) \wedge m = O(A(y^{-1}xy, q)) \vee n$ .

*Remark 4.6:* If A is not (m,n)- upper normal Q- fuzzy soft subgroup of the group G, then above lemma 4.5 is not true.

Example: Let G be the Dihedral group

$G = \{ e, a, b, a^2, ab, ba \}$  and let  $A = \{ (e, 1/5), (a, 4/5), (b, 1/2), (a^2, 4/5), (ab, 4/5), (ba, 4/5) \}$  Since  $O(A(b)) = 1 \neq 2 = O(A(a^{-1}ba))$ .

*Proposition 4.7:* Let G be a finite group and let A,  $\lambda$  be

an(m,n)- upper Q-fuzzy soft subgroups, if  $\lambda \mid m$  and  $A(e, q) \wedge m = \lambda(e, q) \vee n$ . Then  $O(A(x, q)) \wedge m / O(\lambda(x, q)) \vee n$  for all  $x \in G$  such that  $O(\lambda(x, q))$  is finite.

*Proof*

Suppose that  $O(\lambda(x, q)) = k$  , then  $A(e, q) \wedge m = \lambda(e, q) \wedge m = \lambda(x^n, q) \wedge m \leq A(x^n, q) \vee n$  , since  $A(e, q) \wedge m \leq A(x^n, q) \vee n$  and  $A(e, q) \wedge m = A(x^n, q) \vee n$ . Thus,  $O(A(x, q)) / n$  (by lemma 4.3).

*Lemma 4.8:* If A is an (m,n)- upper Q-fuzzy soft subgroup of G, then  $A(e, q) \wedge m \leq A(x, q) \vee n$  for every  $x \in G$ .

*Proof:*  $A(e, q) \wedge m = A((x \cdot x^{-1}), q) \wedge m \leq \max \{ A(x, q), A(x^{-1}, q) \} \vee n = \max \{ A(x, q), A(x, q) \} \vee n = A(x, q) \vee n$ .

*Proposition 4.9:* Let G be a group and A be an (m,n)-upper Q-fuzzy soft subgroup of G. Then for any  $x, y \in G$  such that  $A(x, q) \wedge m \neq A(y, q) \vee n$ , we have  $A(xy, q) \wedge m = \max \{ A(x, q), A(y, q) \} \vee n$

*Proof*

Assume that  $A(y, q) \wedge m > A(x, q) \vee n$ , then  $A(y, q) \wedge m = A((x^{-1}xy), q) \wedge m \leq \max \{ A(x^{-1}, q), A(xy, q) \} \vee n = \max \{ A(x, q), A(xy, q) \} \vee n = A(xy, q) \vee n$

Also A is an (m,n)- upper Q-fuzzy soft subgroup, then :

$A(xy, q) \wedge m \leq \max \{ A(x, q), A(y, q) \} \vee n = A(y, q) \vee n$ . Thus  $A(y, q) \wedge m \leq A(xy, q) \vee n \leq A(y, q) \vee n$  this implies that  $A(xy, q) \wedge m = \max \{ A(x, q), A(y, q) \} \vee n$  The same way if  $A(x, q) \wedge m > A(y, q) \vee n$ .

*Definition 4.4:* A (m,n)- upper Q-fuzzy soft subgroup A of a group G is called (m,n)- upper normal Q-fuzzy soft subgroup if  $A((xyx^{-1}), q) \wedge m \leq A(y, q) \vee n$ , for all  $x, y \in G$ .

*Proposition 4.10* The following conditions are equivalent:

- (I) G is (m,n)- upper normal Q-fuzzy soft subgroup
- (II)  $A(xyx^{-1}, q) \wedge m = A(y, q) \vee n$  for all  $x, y \in G$
- (III)  $A(xy, q) \wedge m = A(yx, q) \vee n$  for all  $x, y \in G$

*Proof*

(I)  $\rightarrow$  (II) :

$A(y, q) \wedge m \geq A(xyx^{-1}, q) \vee n$  for all  $x, y \in G$ , on the other hand we need to show  $A(xyx^{-1}, q) \wedge m \geq A(y, q) \vee n$  for all  $x, y \in G$ .  $A(y, q) \wedge m = A(x^{-1}yx^{-1}x, q) \wedge m \leq A(xyx^{-1}, q) \vee n$ . Then  $A(y, q) \wedge m \leq A(xyx^{-1}, q) \vee n$ , therefore  $A(xyx^{-1}, q) \wedge m = A(y, q) \vee n$  for all  $x, y \in G$ .

(II)  $\rightarrow$  (III) :

Since  $xy = x(yx)x^{-1}$ , then  $A(xy, q) \wedge m = A(x(yx)x^{-1}, q) \wedge m = A(yx, q) \vee n$ , then  $A(xy, q) \wedge m = A(yx, q) \vee n$  for all  $x, y \in G$ .

(III)  $\rightarrow$  (I):

Since  $A(xyx^{-1}, q) \wedge m = A((xy)x^{-1}, q) \wedge m = A(x^{-1}(xy), q) \wedge m = A(x^{-1}xy, q) \wedge m = A(y, q) \vee n$ , then  $A(xyx^{-1}, q) \wedge m \leq A(y, q) \vee n$ , for all  $x, y \in G$

Thus the conditions are equivalent.

*Definition 4.5 :* Let A be an (m,n)- upper Q-fuzzy soft subgroup of a group G. Then the upper Q-fuzzy soft index of A in G is defined by:

The set of the upper Q-fuzzy soft left cosets of A in G, that are themselves are upper Q-fuzzy soft subgroups the set of the upper fuzzy soft left cosets of  $\mu$  in  $G, [G:A] = \text{Card}$  We shall call this type of the upper Q-fuzzy soft index by

general upper Q-fuzzy soft index, because this type is not restricted by any conditions.

*Proposition 4.11:* If A is an (m,n)- upper Q-fuzzy soft subgroup of G, then the value of the upper Q-fuzzy soft index is equal to the number of elements in A which is equal to A (e,q) adding for it one, i.e.,

$$[G : A] = \text{card}(\text{element of A which is equal to A (e,q)} + 1).$$

*Proof*

For all elements of A which is the value of its elements equal to A(e,q). The upper Q-fuzzy left coset of its elements are (m,n)- upper Q-fuzzy soft subgroups; also the upper Q-fuzzy left coset of A (e,q) is also (m,n)- upper Q-fuzzy soft subgroup. And these satisfy the definition of the upper Q-fuzzy index. Therefore:

$$[G : A] = \text{card}(\text{element of A which is equal to A (e,q)} + 1).$$

*Proposition 4.12:* If A is (m,n)- upper Q-fuzzy soft subgroup of a group G, and  $[G : A] = 3$ , then A is an (m,n)- upper normal Q-fuzzy soft subgroup of G.

*Proof*

Suppose that A is an (m,n)- upper Q-fuzzy soft subgroup of G and  $[G : A] = 3$ , this means that A contains two elements having the same value of A(e,q) and let these two elements A (x,q), A (y,q) such that  $x, y \in G$ , we need to prove that A is an (m,n)- upper normal Q-fuzzy soft subgroup, which means that

for all  $z, h \in G$ , we need to show  $A(z h z^{-1}, q) \wedge m \leq A(h, q) \vee n$  or  $A(h z, q) \wedge m = A(z h, q) \vee n$  or  $A(h z h^{-1}, q) \wedge m \leq A(z, q) \vee n$  there exist many cases for A (h,q), A (z,q):

(i) If  $A(h, q) = A(z, q)$  and any of these is not equal to A (e,q), then

$$\begin{aligned} A(z h z^{-1}, q) \wedge m &\leq \max \{A(z h, q), A(z^{-1}, q)\} \vee n \\ &= \max \{A(z h, q), A(z, q)\} \vee n \\ &\leq \max \{\max \{A(z, q), A(h, q)\}, A(z, q)\} \vee n \\ &= \max \{\max \{A(h, q), A(h, q)\}, A(h, q)\} \vee n \\ &= A(h, q) \vee n \end{aligned}$$

$$\text{Then } A(z h z^{-1}, q) \wedge m \leq A(h, q) \vee n.$$

(ii) If  $A(h, q) \neq A(z, q)$  and any of these is not equal to A (e,q)

$A(h z, q) \wedge m = \max \{A(h, q), A(z, q)\} \vee n$  and  
 $A(z h, q) \wedge m = \max \{A(z, q), A(h, q)\} \vee n$  thus  
 $A(z h, q) \wedge m = \max \{A(z, q), A(h, q)\} \vee n = \max \{A(h, q), A(z, q)\} \vee n = A(h z, q) \vee n$ , then  $A(z h, q) \wedge m = A(h z, q) \vee n$ .

(iii) In this case we have two sub cases :

Sub case (a). In the case if the element h of G is the same of x or y or e it follows that  $A(h, q) = A(e, q)$  and the element z of G is different from x, y and e then  $A(e, q) \not\leq A(z, q)$  because there are only two elements in G and their values in A equal to A (e,q), thus  $A(h z h^{-1}, q) \wedge m$

$$\leq \max \{A(h z, q), A(h^{-1}, q)\} \vee n = \max \{A(h z, q), A(h, q)\} \vee n$$

$$\leq \max \{\max \{A(h, q), A(z, q), A(h, q)\} \vee n$$

$$= \max \{A(z, q), A(h, q)\} \vee n$$

$$= A(z, q) \vee n \text{ Then } A(h z h^{-1}, q) \wedge m \leq A(z, q) \vee n.$$

Subcase (b). In the case if the element z of G is the same of x or y or e it follows that  $A(z, q) = A(e)$  and the element h

of G is different from x, y and e then  $A(e, q) \not\leq A(h, q)$  because there are only two elements in G and their values in  $\mu$  equal to  $\mu(e)$ , thus  $A(z h z^{-1}, q) \wedge m \leq \max \{A(z h, q), A(z^{-1}, q)\} \vee n = \max \{A(z h, q), A(z, q)\} \vee n \leq \max \{\max \{A(z, q), A(h, q), A(z, q)\} \vee n$

$$= \max \{A(h, q), A(z, q)\} = A(h, q) \vee n$$

$$\text{Then } A(z h z^{-1}, q) \wedge m \leq A(h, q) \vee n.$$

(iv) if the element h of G is the same of x or y or e it follows that  $A(h, q) = A(e, q)$ , and the element z of G is the same of x or y or e it follows that  $A(z, q) = A(e, q)$  then  $A(z h z^{-1}, q) \wedge m \leq \max \{A(z h, q), A(z^{-1}, q)\} \vee n = \max \{A(z h, q), A(z, q)\} \vee n \leq \max \{\max \{A(z, q), A(h, q)\}, A(z, q)\} \vee n$

$$= \max \{\max \{A(h, q), A(h, q)\}, A(h, q)\} \vee n = A(h, q) \vee n$$

$$\text{Then } A(z h z^{-1}, q) \wedge m \leq A(h, q) \vee n.$$

From the above, all cases that we have studied, we conclude that A is (m,n)- upper normal Q-fuzzy soft subgroup of G.

## 4. Conclusion

We have studied in this paper the definition of the (m,n)- upper Q-fuzzy subgroup over an arbitrary group. Some proposition, theorems and examples given for it and this proposition and corollary are generalization for some properties in group theory.

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## References

- [1] Anthony, J. M. and Sherwood, H. Fuzzy Group Redefined, J. Math. Anal. Appl, 69. 1979), 124 – 130.
- [2] Abd-Allah AM, Omer Rak (Fuzzy Partial Groups. Fuzzy Sets and Systems 82: 1996), 369-374.
- [3] M.I Ali, F. Feng, X.Y. Liu, W. K. Min, M. Shabir On some new operations in soft set theory, Computers and Mathematics with Applications 57, (2009), 1547–1553.
- [4] B. Ahmad and A. Kharal, On Fuzzy Soft Sets, Advances in Fuzzy Systems, Volume 2009
- [5] Dib KA, Hassan AM The Fuzzy Normal Subgroups, Fuzzy. Sets and Systems 98,(1998), 393-402.
- [6] Dong, B: Direct product of anti fuzzy subgroups. J Shaoxing Teachers College 5, 29–34 (1992). in Chinese.
- [7] D. A. Molodtsov, Soft Set Theory - First Result, Computers and Mathematics with Applications, Vol. 37, (1999), pp. 19-31.
- [8] P. K. Maji and A.R. Roy, Soft Set Theory, Computers and Mathematics with Applications 45 (2003), 555 – 562.
- [9] P. K. Maji, R. Biswas and A.R. Roy, Fuzzy Soft Sets, Journal of Fuzzy Mathematics, Vol 9, no.3, (2001), pp.-589-602.

- [10] Rosenfeld, A. Fuzzy groups, J. Math. Anal. Appl, 35. (1971), 512 – 517.
- [11] Shen, Z: The anti-fuzzy subgroup of a group. J Liaoning Normal Univ (Nat Sci) 18(2): (1995). 99–101 in Chinese
- [12] A.Solairaju and R,Nagarajan ,A New structure and constructions of Q- fuzzy group, Advances in Fuzzy Mathematics, Vol.4, No.1 (2009), 23-29.
- [13] A.Solairaju and R,Nagarajan ,Some Structure Properties of Upper Q-fuzzy Index order with upper Q-fuzzy subgroups, International Journal of Open Problems and Applications, Vol.3, No.1(2011), 21-29.
- [14] A.Solairaju and R,Nagarajan Anti Q- fuzzy G-modular distributive lattices, International Journal of Mathematical Archive, Vol.3, No.4(2012), 1-9.
- [15] A.Solairaju and R,Nagarajan On Bipolar Anti Q-fuzzy group, International Journal of Computer Applications, Vol.15,No.6(2010), 30-34.
- [16] G.Subbiah and R.Nagarajan , Degrees of Q-fuzzy group over implication Operator  $[0,1]$ , Elixir Applied Mathematics, Vol.63(2013) 18350-18352
- [17] Yuan, X, Zhang, C, Ren, Y: Generalized fuzzy groups and many-valued implications. Fuzzy Sets Syst. 138, (2003),205–211
- [18] Yao, B:  $(\lambda, \mu)$ -fuzzy normal subgroups and  $(\lambda, \mu)$ -fuzzy quotient subgroups. J Fuzzy Math. 13(3),(2005),695–705.
- [19] Yao, B:  $(\lambda, \mu)$ -fuzzy subrings and  $(\lambda, \mu)$ -fuzzy ideals. J Fuzzy Math. 15(4), (2007),981–987
- [20] Zadeh , L . A. Fuzzy sets , Inform . and Control, 8, (1965),338 – 353.

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