Applied and Computational Mathematics
2015; 4(1-2): 15-19
Published online December 31, 2014 (http://www.sciencepublishinggroup.com/j/acm)
doi: 10.11648/j.acm.s.2015040102.14
ISSN: 2328-5605 (Print); ISSN: 2328-5613 (Online)

Pseudo similar intuitionistic fuzzy matrices

T. Gandhimathi[1], A. R. Meenakshi[2]

[1]Department of Mathematics, P.A. College of Engineering and Technology, Pollachi-642 002, India
[2]Department of Mathematics, Karpagam University, Coimbatore-642 021, India

Email address:
gandhimahes@rediffmail.com (T. Gandhimathi)

To cite this article:
T. Gandhimathi, A. R. Meenakshi. Pseudo Similar Intuitionistic Fuzzy Matrices. *Applied and Computational Mathematics*. Special Issue: New Advances in Fuzzy Mathematics: Theory, Algorithms, and Applications. Vol. 4, No. 1-2, 2015, pp. 15-19.
doi: 10.11648/j.acm.s.2015040102.14

Abstract: In this paper, we shall define Pseudo Similarity and Semi Similarity for Intuitionistic Fuzzy Matrix (IFM) and prove that the Pseudo similarity relation on a pair of IFMs is inherited by all its powers and their transposes are similar. Also we exibit that the Pseudo similarity relation preserve regularity and impotency of their matrices.

Keywords: Fuzzy matrix, Intuitionistic Fuzzy Matrix, Pseudo Similar, Semi Similar

1. Introduction

We deal with fuzzy matrices that is, matrices over the fuzzy algebra FM and FN with support [0,1] and fuzzy operations {+, .} defined as a+b=max{a,b}, a.b = min{a,b} for all a,b$\in$ FM and a + b = min {a, b}, a .b = max{a, b} for all a, b$\in$ FN. A matrix A$\in$ F^{M}_{mxn} is said to be regular if there exists X $\in$ F^{M}_{nxm} such that AXA = A, X is called a generalized inverse (g-inverse) of A. In [4], Kim and Roush have developed the theory of fuzzy matrices, under max min composition analogous to that of Boolean matrices. Cho [2] has discussed the consistency of fuzzy matrix equations. For more details on fuzzy matrices one may refer [6]. Atanassov has introduced and developed the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets [1]. Basic properties of intuitionistic fuzzy matrices as a generalization of the results on fuzzy matrices have been derived by Pal and Khan [5]. In our earlier work, we have discussed on regularity and idempotency of IFMs[8].The Pseudo Similarity in semi groups of fuzzy matrices were discussed by Chen and Meenakshi[3,7]

In this paper, pseudo similarity and semi similarity of an intuitionistic fuzzy matrix are discussed.

2. Preliminaries

In this paper, we are concerned with fuzzy matrices, that is matrices over a fuzzy algebra FM (FN) with support [0,1],under maxmin (minmax) operations and the usual ordering of real numbers. Let $(IF)_{mxn}$ be the set of all intuitionistic fuzzy matrices of order mxn, F^{M}_{mxn} be the set of all fuzzy matrices of order mxn, under the maxmin composition and F^{N}_{mxn} be the set of all fuzzy matrices of order mxn, under the minmax composition.

If A = $(a_{ij}) \in (IF)_{mxn}$, then A = $(\langle a_{ij\mu}, a_{ij\nu}\rangle)$, where $a_{ij\mu}$ and $a_{ij\nu}$ are the membership values and non membership values of a_{ij} in A respectively with respect to the fuzzy sets μ and ν, maintaining the condition $0 \le a_{ij\mu} + a_{ij\nu} \le 1$.

We shall follow the matrix operations on intuitionistic fuzzy matrices as defined in [8].

For $A, B \in (IF)_{mxn}$,then

$$A + B = \left(\left\langle \max\{a_{ij\mu}, b_{ij\mu}\}, \min\{a_{ij\nu}, b_{ij\nu}\}\right\rangle\right)$$

$$AB = \left(\left\langle \max_k \min\{a_{ik\mu}, b_{kj\mu}\}, \min_k \max\{a_{ik\nu}, b_{kj\nu}\}\right\rangle\right)$$

Let us define the order relation on $(IF)_{mxn}$ as ,

$$A \le B \Leftrightarrow a_{ij\mu} \le b_{ij\mu} \text{ and } a_{ij\nu} \ge b_{ij\nu}, \text{for all i and j.}$$

Definition 2.1[5]

An $A \in (IF)_{mxn}$ is said to be regular if there exists

$X \in (IF)_{nxm}$ satisfying AXA = A and X is called a generalized inverse (g-inverse) of A which is denoted by $\bar{A}$.

Let A{1} be the set of all g-inverses of $\bar{A}$.

Definition 2.2[8]

A Matrix $A \in (IF)_n$ is said to be invertible if and only if there exists $X \in (IF)_n$ such that $AX = XA = I_n = \langle I_n^M, I_n^N \rangle$ where I_n is the identity matrix in $(IF)_n$

Definition 2.3[8]

A square intuitionistic fuzzy matrix is called intuitionistic fuzzy permutation matrix, if every row and column contains exactly one <1,0> and all other entries are <0,1>. Let P_n be the set of all nxn permutation matrices in $(IF)_n$.

Definition 2.4[8]

Let $A \in (IF)_n$ .Then

(i) A is reflexive $\Leftrightarrow$ A $\geq I_n$.

(ii) A is symmetric $\Leftrightarrow A = A^T$.

(iii) A is transitive $\Leftrightarrow A^2 \leq A$.

(iv) A is idempotent $\Leftrightarrow A = A^2$

3. Pseudo Similar Intuitionistic Fuzzy Matrices

In this section, the pseudo similarity and semi similarity of an intuitionistic fuzzy matrix are discussed.

Definition 3.1

$A \in (IF)_m$ and $B \in (IF)_n$ are said to be Pseudo-Similar and denote it by $A \simeq B$ if there exist $X \in (IF)_{mxn}$ and $Y \in (IF)_{nxm}$ such that A=XBY ,B=YAX and X=XYX.

Definition 3.2

$A \in (IF)_m$ and $B \in (IF)_n$ are said to be Semi-Similar and denote it by $A \approx B$ if there exist $X \in (IF)_{mxn}$ and $Y \in (IF)_{nxm}$ such that A=XBY and B=YAX.

Definition 3.3

For $A \in (IF)_n$, the least d >0 such that $A^{k+d} = A^k$ for some integer k, is called the period of A.

Theorem 3.1

Let $A \in (IF)_m$ and $B \in (IF)_n$. Then the following are equivalent.

(i) $A \simeq B$

(ii) There exist $X \in (IF)_{mxn}$ and $Y \in (IF)_{nxm}$ such that A=XBY and B=YAX and $XY \in (IF)_m$ is idempotent.

(iii) There exist $X \in (IF)_{mxn}$ and $Y \in (IF)_{nxm}$ such that A=XBY and B=YAX and $YX \in (IF)_n$ is idempotent.

Proof:

(i) $\Rightarrow$(ii) and (i) $\Rightarrow$(iii) are trivial, since X=XYX implies $XY \in (IF)_m$ and $YX \in (IF)_n$ are idempotent matrices.

(ii) $\Rightarrow$(i): A=XBY = (XY)A(XY)= (XYX)B(YXY).

Similarly, B=YAX =(YXY)A(XYX)

Put X'=XYX and Y'= YXY. Then A=X'BY' and B=Y'AX'.

Further using XY is idempotent, we get

X'Y'= (XYX)(YXY) =XY and

(X'Y') (X'Y') =(XY)(XY) = X'Y'.

Thus X'Y' is idempotent.

Set X''=X'Y'X' and Y''= Y'X'Y'. Then

A =X'BY'

= X'(Y'AX') Y'

= X'Y'(X'BY') X'Y'

= (X'Y'X')B(Y'X'Y') = X''B Y''

Similarly, B = Y''A X''.

By using X'Y' is idempotent, we have

X'' Y'' X'' = (X'Y'X') (Y'X'Y') (X'Y'X')

= X'Y'X'

= X''.

Therefore $A \simeq B$.Thus (i) holds.

(iii) $\Rightarrow$(i): This can be proved in the same manner and hence omitted.

Theorem 3.2:

Let $A \in (IF)_m$ and $B \in (IF)_n$. Then the following are equivalent.

(i) $A \simeq B$

(ii) There exist $X \in (IF)_{mxn}$ and $Y \in (IF)_{nxm}$ such that A=XBY and B=YAX and $(XY)^k \in (IF)_m$ is idempotent for some odd $k \in N$.

(iii) There exist $X \in (IF)_{mxn}$ and $Y \in (IF)_{nxm}$ such that A=XBY and B=YAX and $(YX)^k \in (IF)_n$ is idempotent for some odd $k \in N$.

Proof:

(i) $\Rightarrow$(ii): Follows from Theorem (3.1).

(ii) $\Rightarrow$(i): Let k = 2r+1 with $r \in N$,Since A=XBY and B=YAX, we have

A = X(YAX)Y

= (XY) XBY(XY)

= (XY)X(YAX)Y(XY)

= $(XY)^2A(XY)^2$

= $(XY)^2$ XBY $(XY)^2$

Thus proceeding we get,

A = $(XY)^r$ XBY $(XY)^r$.

Similarly, B = $Y(XY)^r A (XY)^rX$.

Set X'= $(XY)^rX$ and Y'=$Y(XY)^r$. Then A=X'BY', B=Y'AX' and X'Y' = $(XY)^rXY(XY)^r$

= $(XY)^{2r+1}$

= $(XY)^k$ is idempotent.

Hence by Theorem (3.1), $A \simeq B$. Thus (i) holds.

(i) $\Rightarrow$(iii): Follows from Theorem (3.1)

(iii) $\Rightarrow$(i): Let k = 2r+1 with $r \in N$,Since A=XBY and B=YAX, we have

A= $X(YX)^rB(YX)^rY$ and

B = $(YX)^rYAX(YX)^r$

Set X'= $X(YX)^r$ and Y'=$(YX)^rY$. Then A=X'BY', B=Y'AX'

and Y'X' = $(YX)^r(YX) (YX)^r$

= $(YX)^{2r+1}$

= $(YX)^k \in (IF)_n$ is idempotent.

Hence by Theorem (3.1), $A \simeq B$. Thus (i) holds.

Theorem 3.3:

Let $A \in (IF)_m$ and $B \in (IF)_n$. Then the following are equivalent.

(i) $A \simeq B$

(ii) There exist $X \in (IF)_{mxn}$ and $Y \in (IF)_{nxm}$ such that

$A=XBY$ and $B=YAX$ and $(XY)\in(IF)_m$ is periodic with even order.
(iii) There exist $X\in(IF)_{mxn}$ and $Y\in(IF)_{nxm}$ such that $A=XBY$ and $B=YAX$ and $(YX)\in(IF)_n$ is periodic with even order.

Proof:
(i)⇒(ii): Follows from Theorem (3.1), since $XY\in(IF)_{mxn}$ is idempotent, it follows that XY is periodic with even order.
(ii) ⇒(i): Suppose A=XBY and B=YAX and XY is periodic with even order2k(k∈N)say, then $(XY)^{2k}=XY$.
Hence $(XY)^{2k-1}(XY)=XY$
Further $(XY)^{2k-1}(XY)^{2k-1} = (XY)^{2k-1}XY(XY)^{2k-1}$
$= (XY)^{2k-1}$.
Thus $(XY)^{2k-1}$ is idempotent.
Therefore A≚B, by Theorem (3.2).Thus (i) holds.
(i)⇔(iii):This can be proved in the same manner and hence omitted.

Theorem 3.4:
Let $A\in(IF)_m$ and $B\in(IF)_n$. Then the following are equivalent.
(i) A≚B
(ii) There exist $X\in(IF)_{mxn}$ and $Y\in(IF)_{nxm}$ such that A=XBY, B=YAX,X=XYX and Y=YXY
(iii) There exist $X\in(IF)_{mxn}$ and $Y,Z\in(IF)_{nxm}$ such that A=XBY, B=ZAX, X=XYX = XZX

Proof:
(i)⇒(iii): Since A≚B, By Definition(3.1),there exist X∈(IF)mxn and Y∈(IF)nxm such that A=XBY, B=YAX,X=XYX. Let Y=Z then B=ZAX and X=XZX as required. Thus(iii) holds.
(iii)⇒(ii): Suppose there exist X∈(IF)mxn and Y,Z∈(IF)nxm such that A=XBY , B=ZAX , X= XYX= XZX , then

$$A=XBY=X(ZAX)Y=(XZX)B(YXY)$$
$$=XB(YXY) \text{ and}$$
$$B=ZAX=Z(XBY)X=(ZXZ)A(XYX)$$
$$=(ZXZ)AX.$$

Set Y'=YXY and Z'=ZXZ. Then

$$X=XYX=XY(XYX)=XY'X \text{ and}$$
$$X=XZX=XZ(XZX)=XZ'X.$$

In addition, we have A=XBY' and B=Z'AX.
Set Y''=Z'XY'. Then XY''X=XZ'(XY'X)=XZ'X=X and Y''XY''=Z'(XY'X)Y'=Z'XY'=Y''.
We check that XBY''=XBZ'XY'

$$=XZ'AXZ'XY'$$
$$=XZ'AXY'$$
$$=XBY'=A,$$

$$Y''AX=Z'XY'XBY'X=Z'XBY'X$$
$$=Z'AX=B.$$

Thus there exist $X\in(IF)_{mxn}$, $Y''\in(IF)_{nxm}$ such that A=XBY'', B=Y''AX, X=XY''X and Y''=Y''XY'' as asserted and (ii) holds.
(ii)⇒(i):This is trivial.

Remark 3.1:
We observe that Theorem (3.4) infers the symmetry of Pseudo similarity relation, that is, A≚B⇔ B≚A.

Theorem 3.5:
Let $A\in(IF)_m$ and $B\in(IF)_n$. Then the following are equivalent.
(i) A≚B
(ii) A^T ≚ B^T
(iii) A^k ≚ B^k for any integer $k\geq1$
(iv) PAP^T ≚ QBQ^T for some permutation matrices $P\in(IF)_m$ and $Q\in(IF)_n$

Proof:
(i)⇔(ii): This is direct by taking transpose on both sides of A=XBY,B=YAX and using $(A^T)=A$ and $(AX)^T=X^TA^T$.
(i)⇔(iii): (iii)⟹(i) is trivial. (i)⟹(iii) can be proved by induction on k. Since A≚B, there exist $X\in(IF)_{mxn}$ and $Y\in(IF)_{nxm}$ such that A=XBY,B=YAX,X=XYX .By Theorem(3.4),further we have Y=YXY. Let us assume that $A^r=XB^rY$, $B^r=YA^rX$ for some r∈N.

$$A=XBY\Longrightarrow AXY=XBYXY$$
$$=XBY \quad =A,$$

$$XB^{r+1}Y = XB^rBY$$
$$= XB^rBY$$
$$= XB^r(YAXY)$$
$$= XB^rYA \quad = A^{r+1}$$

Similarly it can be checked YAr+1X = Br+1.
Thus by induction for all positive integer k ,we get, Ak=XBkY, Bk=YAkX for X∈(IF)mxn and Y∈(IF)nxm such that X=XYX. Hence Ak ≚Bk.
Thus (i) ⟹(iii) hold.
(i)⇔(iv): Suppose A≚B, then there exist X∈(IF)mxn and Y∈(IF)nxm such that
A=XBY, B=YAX,X= XYX.
$A=XBY\Longrightarrow PAP^T= PXBYP^T$

$$= (PXQ^T)(QBQ^T)(QYP^T)$$

$B=YAX \Longrightarrow QBQ^T= QYAXQ^T$

$$= (QYP^T)(PAP^T)(PXQ^T).$$

Set $X'=PXQ^T$, $Y'=QYP^T$ then X'Y'X'=X', $PAP^T=X(QBQ^T)Y'$ and $QBQ^T=Y'(PAP^T)X'$.
Thus PAP^T ≚ QBQ^T. Conversely, if PAP^T ≚ QBQ^T then as above,
$P^T(PAP^T)P$ ≚ $Q^T(QBQ^T)Q \Longrightarrow A$ ≚B
Thus (i)⇔(iv) holds.

Theorem 3.6:

Let $A\in(IF)_m$ and $B\in(IF)_n$. Then the following are equivalent.

(i) $A\approx B$

(ii) $A^T\approx B^T$

(iii) $A^k\approx B^k$ for any integer $k\geq 1$

(iv) $PAP^T\approx QBQ^T$ for some permutation matrices $P\in(IF)_m$ and $Q\in(IF)_n$

Proof:

This can be proved along the same manner as that of Theorem (3.5) and hence omitted.

Remark 3.2:

It is clear that Similarity ⇒ Pseudo similarity ⇒ Semi similarity but converse is not true.

Example 3.1:

Consider the IFMs,

$$A=\begin{bmatrix} <0.5,0> & <0.5,0> \\ <0,1> & <0,1> \end{bmatrix}$$

and

$$B=\begin{bmatrix} <0.5,0> & <0,1> \\ <0.5,0> & <0,1> \end{bmatrix}$$

with respect to

$$X=\begin{bmatrix} <1,0> & <0,1> \\ <0,1> & <0,1> \end{bmatrix}$$

and

$$Y=\begin{bmatrix} <1,0> & <1,0> \\ <1,0> & <0,0.5> \end{bmatrix}\in X\{1\}.$$

Here A=XBY and B=YAX.

This implies that $A\backsimeq B$ and $A\approx B$.

Example 3.2

Let us consider the IFMs

$$X=\begin{bmatrix} <0.3,0.3> & <1,0> \\ <0.5,0.5> & <0.2,0.2> \end{bmatrix}$$

and

$$Y=\begin{bmatrix} <1,0> & <1,0> \\ <0,0.5> & <0,0.5> \end{bmatrix}$$

then $XYX\neq X$ and $YXY\neq Y$.

$$\text{For } A=\begin{bmatrix} <0.5,0.3> & <0.2,0.3> \\ <0,0.5> & <0,0.5> \end{bmatrix}$$

and

$$B=\begin{bmatrix} <0.3,0.3> & <0.3,0> \\ <0.5,0.5> & <0.5,0.5> \end{bmatrix},$$

B=XAY and A=YAX.

Hence $A\approx B$ but $A\backsimeq B$.

Next we can see that Pseudo Similarity relation preserve regularity and idempotency of matrices concerned.

Theorem 3.7:

Let $A\in(IF)_m$ and $B\in(IF)_n$ such that $A\backsimeq B$. Then A is regular matrix ⇔ B is regular matrix.

Proof:

Since $A\backsimeq B$,by Theorem(3.4), there exist $X\in(IF)_{mxn}$ and $Y\in(IF)_{nxm}$ such that A=XBY, B=YAX,X=XYX and Y=YXY. Suppose A is regular, then there exists $G\in(IF)_m$ such that AGA=A. DefineU=YGX. Clearly $U\in(IF)_n$.Since,

$$\begin{aligned} AXY&=XBYXY = XBY=A\\ &=(XYX)BY\\ &=XYA,\\ BUB &= (YAX)(YGX)(YAX)\\ &=Y(AXY)G(XYA)X\\ &= YAGAX\\ &=YAX=B. \end{aligned}$$

Hence B is regular.Converse can be proved in the same manner.

Theorem 3.8:

Let $A\in(IF)_m$ and $B\in(IF)_n$ such that $A\backsimeq B$. Then A is idempotent ⇔ B is idempotent.

Proof:

Since $A\backsimeq B$,by Theorem(3.4), there exist $X\in(IF)_{mxn}$ and $Y\in(IF)_{nxm}$ such that B=YAX and A=XBY⇒AXY=A=AYX.

Suppose A is idempotent, then $A^2=A$, hence

$$\begin{aligned} B^2&=(YAX)(YAX)=Y(AXY)AX\\ &= YAAX\\ &=YA^2X=YAX=B \text{ and B is idempotent.} \end{aligned}$$

Converse can be proved in the same manner.

4. Conclusion

In this paper, the Pseudo similarity and Semi similarity of IFMs are defined .The Pseudo similarity relation on a pair of IFMs is inherited by all its powers and the Pseudo similarity relation preserve regularity and impotency of matrices are discussed.

References

[1] K. Atanassov, “Intuitionistic fuzzy sets”, Fuzzy Sets and systems 20(1), (1986), 87-96.

[2] H. H. Cho, “Regular fuzzy matrices and fuzzy equations”, Fuzzy Sets and Systems, 105 (1999), 445 – 451.

[3] Huanyin Chen, “Pseudo similarity in Semigroups”, Semi group forum,68(2004),59-63

[4] K. H. Kim and F. W. Roush, “Generalised fuzzy matrices”, Fuzzy sets and Systems, 4 (1980),293 – 315.

[5] S. Khan and Anita Pal, “The Generalised inverse of intuitionistic fuzzy matrics”, Journel of Physical science, Vol II (2007), 62-67.

[6] A R. Meenakshi,"Fuzzy matrices: Theory and its applications", MJP Publishers, Chennai – 2008.

[7] A R. Meenakshi," Pseudo similarity in semigroups of fuzzy matrices", Proc. Int. Symp. on Semigroups and Appln.,(2007),64-73.

[8] A R. Meenakshi and T. Gandhimathi, "On Regular intuitionistic Fuzzy matrices", Int.J. of Fuzzy Mathematics, Vol. 19, No. 2, (2011), 599-605.