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# Domination Number and Secure Resolving Sets in Cyclic Networks

**Basma Mohamed, Mohamed Amin**

Mathematics and Computer Science Department, Faculty of Science, Menoufia University, Shebin El-Kom, Egypt

## Email address:

bosbos25jan@yahoo.com (Basma Mohamed)

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**Abstract:** Consider a robot that is navigating a graph-based space and is attempting to determine where it is right now. To determine how distant it is from each group of fixed landmarks, it can send a signal. We discuss the problem of determining the minimum number of landmarks necessary and their optimal placement to ensure that the robot can always locate itself. The number of landmarks is referred to as the graph's metric dimension, and the set of nodes on which they are distributed is known as the graph's metric basis. On the other hand, the metric dimension of a graph $G$ is the minimum size of a set $w$ of vertices that can identify each vertex pair of $G$ by the shortest-path distance to a particular vertex in $w$. It is an NP-complete problem to determine the metric dimension for any network. The metric dimension is also used in a variety of applications, including geographic routing protocols, network discovery and verification, pattern recognition, image processing, and combinatorial optimization. In this paper, we investigate the exact value of the secure resolving set of some networks, such as trapezoid network, $Z$-$(P_n)$ network, open ladder network, tortoise network and $P_{2n}\bar{V}P_n$ network. We also determine the domination number of the networks, such as the twig network $T_m$, double fan network $F_{2,n}$, bistar network $B_{n,n}$ and linear $kc_4$ – snake network.

**Keywords:** Domination Number, Secure Resolving Set, Twig Graph and Linear *kc4* - Snake Graph

## 1. Introduction

All of the graphs $G$ considered are finite, undirected and have many edges without loops. A subset $S = \{u_1, u_2,..., u_k\}$ of vertex set $V(G)$ is called a resolving set if, for any vertex $x \in V(G)$, the code of $x$ with regard to $S$, indicated by $C_S(x)$, which is defined as $C_S(x) = (d(u_1, x), d(u_2, x),.., d(u_k, x))$, is different for distinct $x$. The dimension of $G$, represented by dim $(G)$, is the minimal cardinality of a resolving set. Slater [1] first proposed the idea of metric dimension, which Harary and Melter [2] investigated separately. Since then, a lot of research has been done on this problem. In several fields of science and technology, the metric dimension has several uses.

Security is a concept that is linked to a number of different types of sets in a graph. For example, a dominating set $D$ of $G$ is secure if there exists $u \in D$ such that $(D - \{u\}) \cup \{v\}$ is a dominating set for any $v \in V - D$ [2, 3]. Subramanian et al. [4] introduced secure resolving sets and secure resolving dominating sets for several classes of graphs. The study of domination is the fastest developing topic in graph theory, owing to its numerous and diverse applications in domains such as social sciences, communications networks, algorithmic designs, and so on. Berge [5] in 1958 and Ore [6] in 1962 gave rigorous mathematical definitions to the problem of domination. Berge used the terms "external stability" and "domination number of external stability coefficient" to describe the domination. Domination theory has several applications in wireless communication networks [7], business networks, and decision-making. Khalil [8] demonstrated the domination numbers for the helm graph $H_n$ and the web graph $W_n$. Nagabhushana et al. [9] demonstrated the domination number for the friendship graph $F_n$ and the windmill graph $Wd(m,n)$. Murthy [10] demonstrated the dominance number for the tadpole graph $T_{m,n}$. Kavitha et al. [11] demonstrated the dominance number for the book graph $B_n$ and stacked book graph $B_{3,n}$. Sugumaran et al. [12]

discussed the dominating set and domination number of the graphs such as fan $F_{m,2}$, diamond snake $D_n$, banana tree $B(m, n)$, coconut tree $CT(m, n)$, firecracker $F(m, n)$.

Our main aim in this paper is to compute the secure resolving set of some graphs, including the trapezoid graph, the $Z$-$(P_n)$ graph and the open ladder graph, the tortoise graph, $P_{2n} \triangledown P_n$ *graph* , We also determine the domination number of the graphs, such as the twig graph $T_m$, the double fan graph $F_{2,m}$, the bistar graph $B_{n,n}$ and the linear $kc_4$ - snake graph.

## 2. Preliminaries

We present some definitions and known results in this section that are needed to prove our main theorems.

Definition 2.1 [13]: Trapezoid graphs $T_n$ are intersection graphs of trapezoids between two horizontal lines. Interval graphs and permutation graphs are subclasses of this subset of co-comparability graphs. A graph is called a trapezoid graph if there exists a set of trapezoids corresponding to the vertices of the graph such that two vertices are connected by an edge if and only if the corresponding trapezoids intersect.

Definition 2.2 [$(Z$-$(P_n)$] [14]: In a pair of paths $P_n$ $i^{th}$ vertex of path $P_1$ is connected to the $i+1^{th}$ vertex of path $P_2$. It is denoted by $Z$-$(P_n)$.

Definition 2.3 [15]: An open ladder $O(L_n)$, $n \geq 2$ is generated from two paths of length $n$-1 with $V(G) = \{u_i, v_i: 1 \leq i \leq n\}$ and $E(G) = \{u_i u_{i+1}, v_i v_{i+1}: 1 \leq i \leq n-1\} \cup \{u_i v_i: 2 \leq i \leq n-1\}$.

Definition 2.4 Twig graph [16]: A graph $G(V,E)$ derived from a path by adding precisely two pendant edges to each of the internal vertices of the path is called a twig. A twig $T_m$ with "$m$" internal vertices has $3m$+1 edges and $3m$+2 vertices.

Definition 2.5 [17]: The double fan $DF_n$ is made up of two fan graphs that share a path. In other terms $DF_n = P_n + \overline{K_2}$.

Definition 2.6 [18]: Bistar $B_{n,n}$ is the graph obtained by connecting the middle (apex) vertices of two copies of $K_{1,n}$ by an edge.

Definition 2.7 [19]: A tortoise $G(T_n)$ is obtained from a path $v_1, v_2, \ldots, v_n$ by adding an edge between $v_i$ and $v_{n-i+1}$ for i= 1 to $\left\lfloor \frac{n}{2} \right\rfloor$ and $n \geq 3$.

Definition 2.8 [20]: The join graph $P_{2n} \triangledown P_n$ with $3n$ vertices, is comprised of a simple path $P_{2n}$ with $2n$ vertices, $u_1, u_2, \ldots, u_{2n}$ and a null graph $N_n$ with $n$ vertices, $v_1, v_2, \ldots, v_n$ such that $v_n \in N$ adjacent with $u_1$ and $u_{2n}$ in $P_{2n}$ and $v_{n-1} \in N_n$ adjacent with $u_2$ and $u_{2n-1}$ in $P_{2n}$ and so on.

## 3. Secure Resolving Dimension

In this section, our goal is to find the secure resolving set of some special graphs such as trapezoid graph $T_n$ , $Z$-$(P_n)$ graph, open ladder graph, tortoise graph and $P_{2n} \triangledown P_n$ *graph* .

Theorem 3.1: let $G$ is a trapezoid graph $T_n$ with $k$ blocks and $n$ vertices, then $Sdim(T_n) = 2$.

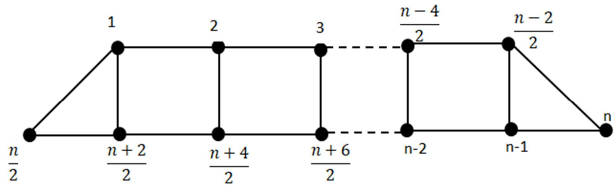


***Figure 1.*** *Trapezoid graph* $T_n$ .

Proof:

We label the trapezoid graph $T_n$ as shown in Figure 1. It is clear that the number of vertices is $n=2k+4$ such that $k$ is the number of blocks of $G$. *let* $S = \{v_1, v_n\}$

Begin

for (i=1; i $\leq \frac{n}{2} - 1$; i++) do

$j_1 = 0, j_2 = k+1$

$d(v_i, S) = (j_1, j_2)$

$j_1 = j_1+1, j_2 = j_2-1$

end

$d(v_{\frac{n}{2}}, S) = (1, \frac{n}{2})$

for ($i = \frac{n}{2} + 1$; $i \leq n$; $i$++)

$j_1=1$

$d(v_i, S) = (j_1, n-i)$

$j_1 = j_1 + 1$

end

end

This completes the proof.

The algorithm of the proof of Theorem 3.1 includes two for-loops, but they are not inner loops; therefore, the method complexity is O ($n$).

Theorem 3.2: let $G$ be $Z$-$(P_n)$ graph with $k$ blocks and $n$ vertices, then $Sdim$ ($Z$-$(P_n)$) = 2.

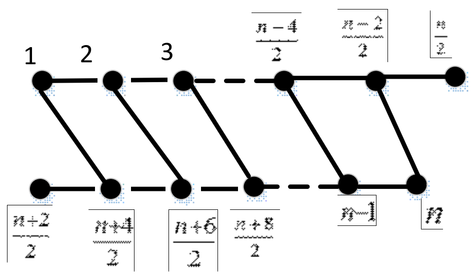


***Figure 2.*** *Z-($P_n$) graph.*

Proof: We label the $Z$-$(P_n)$ graph as shown in Figure 2. It is clear that the number of vertices is $n=2k+4$ such that $k$ is the number of blocks of $G$. let $w = \{v_1, v_{k+3}\}$ .

Begin

for (i=1; i $\leq \frac{n}{2} - 1$; i++) do

$j_1 = 0, j_2 = k+1$

$d(v_i, S) = (j_1, j_2)$

$j_1 = j_1+1, j_2 = j_2-1$

end

$d(v_{\frac{n}{2}}, S) = (1, \frac{n}{2})$

for (i= $\frac{n}{2} + 1$; i $\leq$ n; i++)

$j_1=1$

$d(v_i, S) = (j_1, n-i)$
$j_1 = j_1 + 1$
end
end

This completes the proof.

The algorithm of the proof of Theorem 3.2 includes two for-loops, but they are not inner loops; therefore, the method complexity is O($n$).

Lemma 3.3: let $G$ be an open ladder graph with $k$ blocks and $n$ vertices, then $sdim$ ($O(Ln)$)= 2.

Theorem 3.4: If $G$ is a tortoise graph with $n$ vertices, then $Sdim$ ($T_n$) = 2.

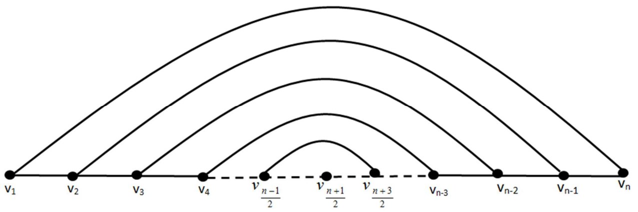


***Figure 3.*** *Tortoise graph $T_n$.*

Proof: We label the $T_n$ graph as shown in Figure 3. It is clear that the number of vertices is $n$. *let* $w = \{v_1, v_n\}$
Begin
for (i=;$1 \le \frac{n}{2}$; i++) do
$d(v_i, w) = (i-1, i+1)$
end
$d(v_{\frac{n}{2}+1}, w) = (2, 0)$
for (i=$n-k$ ; i$\le n$ ; i++) do
$j_1$=1, $j_2$=1
$d(v_i, w) = (j_1, j_2)$
$j_1 = j_1 + 1, j_2 = j_2 + 1$
end
end

This completes the proof.

The algorithm of the proof of Theorem 3.4 includes two for-loops, but they are not inner loops; therefore, the method complexity is O ($n$).

Theorem 3.5: let $G$ is $P_{2n} \,\overline{\vee}\, P_n$ graph with $n$ vertices, then $Sdim$ $(P_{2n} \,\overline{\vee}\, P_n) = 2$

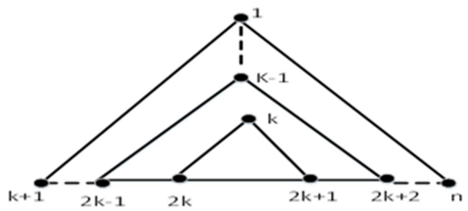


***Figure 4.*** *$P_{2n} \,\overline{\vee}\, P_n$ graph .*

Proof: We label $P_{2n} \,\overline{\vee}\, P_n$ graph as shown in Figure 4. It is clear that the number of vertices is $n$ and $k$ is the blocks number. It is clear that $|V(G)|$ is $n$ = 3k. *let* $w = \{v_1, v_{2k+1}\}$
Begin
for (i=1; $i \le \frac{n-1}{2}$; $i++$) do
$d(v_i, w) = (i-1, i)$
end
$d(v_{\frac{n+1}{2}}, w) = (i-1, i-1)$
for ($i = \frac{n+3}{2}$; $i \le n$ ;$i$++) do
$d(v_i, w) = (n-i+1, n-i)$
end
end

This completes the proof.

The algorithm of the proof of Theorem 3.5 includes three for-loops, but they are not inner loops; therefore, the method complexity is O ($n$).

# 4. Domination Number

Definition 4.1: If every vertex in $V$ - $D$ is neighboring to some vertex in $D$, the set $D$ of vertices in a graph $G$ is called a dominating set. The minimum cardinality of the dominating set of $G$ is the dominance number $\gamma(G)$.

In this section, we propose some identities relating to the dominance number of the twig graph $T_m$, the double fan graph $F_{2,n}$, the bistar graph $B_{n,n}$ and the linear $kC_4$ - snake graph.

Theorem 4.2: The domination number of the twig graph $T_m$ formed from $P_{m+2}$ is $m$
(Where $m = \frac{n-2}{3}$ and $n$ is the number of vertices)
Proof:
Let $G \cong T_m$ be a twig graph on $3m+2$ vertices with ($3m+1$) edges and let $D$ be the minimum dominating set of graph $G$. By definition, the twig graph is derived from a path by adding precisely two pendant edges to each of the internal vertices of the path.

Twig formed from the path with ($m+2$) vertices $T_m$, if we choose all of the central vertices as a single set, it will dominate all the other vertices of $G$. So we will get a minimum dominating set and its cardinality is the domination number of graph $G$. Therefore, the domination number of $G$ is $m$, that is, $\gamma(G) = m$.

*Theorem 4.3*: The domination number of a double fan graph $F_{2,m}$ is 2, where $m \ge 1$ and $m$ is the number of blocks.

*Proof*: Let $G \cong F_{2,m}$ be a fan graph on $m$+ 3 vertices with 3$m$ +2 edges and let $D$ be the minimum dominating set of graph $G$. By definition of the double fan graph, the graph
$DF_n = P_n + \overline{K_2}$.
consists of two fan graphs that have a common path.

There are $m$+1 nodes available in path $P_{m+1}$ of the fan graph, so if we choose any one vertex from path $P_{m+1,}$ then all the other vertices of $G$ are dominated by our chosen vertex. So we will get a minimum dominating set and its cardinality is the domination number of graph $G$.

Hence the dominating set $D$ of $G$ = {1,$m$+3}

Therefore, the domination number of graph $G$ is 2. That is, $\gamma(G) = 2$.

*Theorem 4.4*: The domination number of Bistar $B_{n,n}$ is 2, where $n \geq 2$ and $n$ is the number of vertices.

*Proof*: Let $G \cong B_{n,n}$ be a bistar graph $B_{n,n}$ with $2(n+1)$ vertices and $2n+1$ edges and let $D$ be the minimum dominating set of graph $G$. By definition of the bistar $B_{n,n}$, the graph is obtained by connecting the middle (apex) vertices of two copies of $K_{1,n}$ by an edge.

If we choose all of the center vertices as a single set, it will dominate all the other vertices of

$G$. So we will get a minimal dominating set, and its cardinality is the domination number of graph $G$.

Therefore, the domination number of $G$ is 2. That is, $\gamma(G) = 2$.

*Lemma 4.5*: The domination number of linear $kC_4$ - snake graph is $k+1$, where $k \geq 1$ and $k$ is the number of blocks.

# 5. Conclusion

Domination in graphs is a branch of graph theory that has received a lot of attention. In this paper, we have presented some theorems of the secure resolving set of some special graphs such as the trapezoid graph, $Z$-$(P_n)$ graph, the open ladder graph, the tortoise graph $T_n$, $P_{2n} \,\overline{\vee}\, P_n$ *graph* . Also, we presented some theorems related to domination number in various families of graphs.

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