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ON (m, n) –upper Q-fuzzy soft subgroups

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Abstract: In this paper we shall study some properties for upper Q- fuzzy subgroups, some lemma and theorem for this subject. We shall study the upper Q- fuzzy index with the upper fuzzy sub groups; also we shall give some new definitions for this subject. On the other hand we shall give the definition of the upper normal fuzzy subgroups, and study the main theorem for this. We shall also give new results on this subject.

Keywords: Fuzzy Set, Soft Set, Fuzzy Soft Set, (m, n) –Upper Q-Fuzzy Soft Group, Product, Upper Q-Fuzzy Order, Upper Q-Fuzzy Cossets, Upper Q-Fuzzy Index

1. Introduction

Fuzzy sets was first introduced by Zadeh [20] and then the fuzzy sets have been used in the reconsideration of classical mathematics. Yuan et al. [17] introduced the concept of fuzzy subgroup with thresholds. A fuzzy subgroup with thresholds l and μ is also called a (λ, μ)-fuzzy subgroup. Yao continued to research (λ, μ)-fuzzy normal subgroups, (λ, μ)-fuzzy quotient subgroups and (λ, μ)-fuzzy sub rings in [18,19]. Shen researched anti-fuzzy subgroups in [11] and Dong [6] studied the product of anti-fuzzy subgroups. Molodtsov [7] introduced the concept of soft sets that can be seen as a new mathematical theory for dealing with uncertainty. Molodtsov applied this theory to several directions [7], and then formulated the notion of soft number, soft derivative, soft integral, etc in [8] , the soft set theory has been applied to many different fields with great success. Maji et.al [8] worked on theoretical study of soft sets in detail presented an application of soft set in the decision making problem using the reduction of soft sets.

Majiet.al [9] presented the concept of fuzzy soft sets by embedding the ideas of fuzzy sets. By using this definition of fuzzy soft sets many interesting applications of soft set theory have been expanded by some Researchers. Roy and Maji [9] gave some applications of fuzzy soft sets. A.Solairaju and R.Nagarajan introduced the concept of Structures of Q- fuzzy groups [12]. A.Solairaju and R.Nagarajan studied some structure properties of upper Q- fuzzy index order with upper Q- fuzzy subgroups[13].In this paper, We analyze the notions of (m, n)-Upper Q- fuzzy subgroups, studied some properties for upper Q- fuzzy subgroups, some propositions and theorem for this subject and discussed the product of them.

2. Preliminaries

This section, briefly reviews the basic characteristics of fuzzy set and fuzzy soft sets.

Definition 2.1: [19] By a fuzzy subset of a nonempty set X we mean a mapping from X to the unit interval [0,1].

Definition2.2: [11] If A is a fuzzy subset of X, then we denote A(a) = {x ε X|A(x) ≤ α} for all α ε [0,1].

Throughout this article, we will always assume that

$$0 \leq m < n \leq 1.$$

Let G, G_1, and G_2 always denote groups in the following. I, I_1, and I_2 are identities of G, G_1, and G_2, respectively.

Definition 2.3:[7] Let U be an initial universe, P (U) be the power set of U, E be the set of all parameters and A⊆ E. A soft set (f_A, E) on the universe U is defined by the set of order pairs

$$(f_A, E) = \{(e, f_A(e)) : e \in E, f_A \in P(U)\} \text{ where } f_A : E \rightarrow P(U) \text{ such that } f_A(e) = \phi \text{ if } e \notin A.$$

Here f_A is called an approximate function of the soft set.

Example: Let U = $\{u_1, u_2, u_3, u_4\}$ be a set of four shirts and E = {white(e_1),red(e_2),blue (e_3)} be a set of parameters. If A = $\{e_1, e_2\}$ ⊆ E , $f_A(e_1) = \{u_1, u_2, u_3, u_4\}$ and $f_A(e_2)$=

$\{u_1,u_2,u_3\}$, then we write the soft set (f_A , E)= $\{(e_1, \{u_1, u_2 ,u_3, u_4\}), (e_2,\{u_1,u_2,u_3\})\}$ over U which describe the "colour of the shirts" which Mr. X is going to buy.

Definition 2.4:[9] Let U be an initial universe, E be the set of all parameters and A⊆ E. A pair (F, A) is called a fuzzy soft set over U where F: A → P (U) is a mapping from A into P (U) , where P (U) denotes the collection of all fuzzy subsets of U.

Example: Consider the above example, here we cannot express with only two real numbers 0 and 1, we can characterized it by a membership function instead of crisp number 0 and 1, which associate with each element a real number in the interval [0,1].Then

$(f_A$, E) = $\{\{f_A(e_1)$ = $\{(u_1$, 0.7), (u_2, 0.5), (u_3, 0.4), (u_4, 0.2)$\}$,

f_A (e_2) = $\{(u_1$, 0.5), $(u_2$, 0.1), $(u_3$, 0.5)$\}\}$ is the fuzzy soft set representing the "colour of the shirts" which Mr. X is going to buy.

Definition2.5: A fuzzy soft set A of a group G is called (m,n)- Upper Q-fuzzy soft subgroup of G if for all x,y z ε G, [UFSG1] A(xy,q) ∧ m ≤ max {A(x,q), A(y,q)} ∨ .n and [UFSG2] A(x^{-1},q) ∧ m ≤ A(x,q) ∨ . n where x^{-1} is the inverse element of x.

Throughout this article, max {A(x,q), A(y,q)} is sometimes replaced as A(x,q) ∨ . A(y,q).

Proposition-1: If A is (m,n) –upper Q-fuzzy soft group of G, then A(1,q) ∧ m ≤ A(x,q) ∨ .n for all x ε G, when 1 is the identity of G.

Proof: For all x εG and let x^{-1} be the inverse element of x. Then A(1,q) ∧ m = A(xx^{-1},q) ∧ m= (A(xx^{-1},q) ∧ m) ∧ m ≤ max { A(x,q) , A(x^{-1},q)} ∨ . n = max { A(x,q) ∧ m , A(x^{-1},q) ∧ m} ∨ . n(n ∨ .m) = A(x,q) ∨ . Un.

3. Main Results

Theorem-3.1: Let A be upper Q-fuzzy soft subset of a group G.Then A is (m,n)-upper Q-fuzzy soft group of G if and only if A(x^{-1}y,q) ∧ m ≤ max{A(x,q), A(y,q)} ∨ .n for all x,y ε G.

Proof: Let A is (m,n)-upper Q-fuzzy soft group of G.Then

A(x^{-1}y,q) ∧ m = A(x^{-1}y,q) ∧ m ∧ m ≤ (max{A(x^{-1},q),A(y,q)} ∨ .n) ∧ m

= (A(x^{-1},q) ∧ m, A(y,q)) ∨ . (n ∧ m) ≤ ((A(x,q) ∨ .n) V (A(y,q))) ∨ .n

= max {A(x,q), A(y,q)} ∨ .n.

Conversely, suppose A(x^{-1}y,q) ∧ m ≤ max {A(x,q), A(y,q)} ∨ .n for all x,y ε G, then A(1,q)∩m = A(x^{-1}x,q) ∧ m ≤ max {A(x,q), A(y,q)} ∨ .n =A(x,q) ∨ .n.

So

(A(x^{-1},q) ∧ m =(A(x^{-1}.1,q) ∧ m =(A(x^{-1},q) ∧ m ∧ m ≤ max {A(x^{-1},q), ∨ A(y,q)} ∨ .n) ∧ m

=(A(x^{-1},q) ∧ m) ∨ . (A(y,q) ∨ .n) ∧ ≤ max{(A(x,q) ∨ .n), (A(y,q)} ∨ .n)}

= max{A(x,q), A(y,q)} ∨ n. So A is a (m,n)-upper Q-fuzzy soft group of G.

Theorem-3.2: Let A be a Q-fuzzy soft subset of a group. Then the following are equivalent

(i). A is (m,n)-upper Q-fuzzy soft subgroup of G.(ii)A(α) is a subgroup of G, for any αε [m,n], where A(α) ≠Φ.

Proof: "(i) implies (ii)"

Let A be [m,n]-upper Q-fuzzy soft subgroup of G. For any αε [m,n],such that A(α) ≠Φ,we need to show that x^{-1}yε A(α),for all x,y ε A(α).Since A(x,q) ≤ α and A(y,q) ≤ α, then

(A(x^{-1},q) ∧ m ≤ max{(A(x,q) ∨ .n), (A(y,q)} ∨ n)}≤ max{α,α} ∨ .n =αVn=α. Note that α ≤ n, we obtain (A(x^{-1}y,q)≤ α.so x^{-1}y ε A(α).

"(ii) implies (i)"

Conversely,let A(α) is a subgroup of G. we need to prove that (A(x_1,q) ∧ m≤ max{(A(x,q) ∨ .n),for all x ε G. If there exist x_o,y_o ε G such that A($x_o^{-1}y_o$,q) ∧ m = α ≥ max { A(x_o,q), A(y_o,q)} ∨ .n, thenA(x_o,q) ≤ α, A(y_o,q)≤ α and αε [m, n].Thus x_oε A(α).

And y_oε A(α). But thenA($x_o^{-1}y_o$,q)) ≥α, that is $x_o^{-1}y_o$ does not belong to A(α).This is a contradiction with that A(α) is a subgroup of G. Hence A(x^{-1}y,q) ∧ m≤ max{(A(x,q) ∨ .n), (A(y,q)} ∨ .n holds for any x,y εG.Therefore A is a (m,n) – upper Q-fuzzy soft subgroup of G. we getΦ = 1, where Φ is the empty set.

Theorem-3.3: Let f: G_1→G_2 be a homomorphism and let A be (m.n)-upper Q-fuzzy soft subgroup of G_1.Then f(A) is (m,n)-upper Q-fuzzy soft subgroup of G_2, where f(A)(δ,q) = inf {A(x,q) ∣ f(x) = δ}, for all δεG_2 and x εG.

Proof: If $f^{-1}(y_1)$= Φ or $f^{-1}(y_1)$= Φ for any y_1,y_2 ε G_2, then (f(A)($\delta_1^{-1}\delta_2$,q) ∧ m ≤ 1 = max {f(A)(δ_1,q), f(A)(δ_2,q)} ∨ .n.

Suppose that $f^{-1}(y_1)$= Φ or $f^{-1}(y_1)$= Φ for any y_1,y_2 ε G_2. Then For any y_1,y_2 ε G_2, we have

(f(A)($\delta_1^{-1}\delta_2$,q) ∧ m = inf {A(t,q) ∣ f(t) = $\delta_1^{-1}\delta_2$}∩m as tεG.

= inf {A(t,q) ∣ f(t) = $\delta_1^{-1}\delta_2$} ∧ m as tεG_1≤ inf {A($x_1^{-1}x_2$,q) ∧ m ∣ f(x_1) = δ_1, f(x_2) = δ_2} as x_1,x_2 ε G_1.

≤ inf {max{A(x,q), A(y,q)} ∨ .n ∣ f(x_1) = δ_1, f(x_2) = δ_2} as x_1,x_2 ε G_1.

≤ inf {max A(x_1,q) ∣ f(x_1) = δ_1}, inf {max A(x_2,q) ∣ f(x_2) = δ_2}}

= max {f(A)(δ_1,q), f(A)(δ_2,q)} ∨ .n

So f(A) is (m,n)-upper Q-fuzzy soft subgroup of G_2.

Theorem-3.4: Let f: G_1 →G_2 be a homomorphism and let B be (m,n)-upper Q-fuzzy soft subgroup of G_2. Then f^{-1}(B) is (m,n)- upper Q-fuzzy soft subgroup of G1, where f^{-1}(B)(x,q) = B(f(x,q)), for all x εG_1.

Proof: For any x_1,x_2 ε G_1.

f^{-1}($x_1^{-1}x_2$,q) ∧ m = B(f($x_{1-1}x_2$,q)) ∧ m

= B((f(x_1,q))$^{-1}$ f(x_2,q)) ∧ m ≤ max{B(f(x_1,q), B(f(x_2,q)) ∨ .n

$= \max \{f^{1}(B)(x_1,q), f^{1}(B)(x_2,q)\} \vee .n$

So, $f^{1}(B)$ is (m,n)-upper Q-fuzzy subgroup of G_1.

Let G_1 be a group with the identity I_1 and G_2 be a group with the identity I_2, then $G_1 \times G_2$ is a group with the identity (I_1,I_2) if we define $(x_1,y_1)(x_2,y_2) = (x_1x_2, y_1y_2)$ for all (x_1,y_1), (x_2,y_2) ε $G_1 \times G_2$. Moreover, the inverse element of any (x,a) ε $G_1 \times G_2$ is (y, b) ε $G_1 \times G_2$ if and only if y is the inverse element of x in G_1 and b is the inverse element of a in G_2.

Theorem-3.5:Let A and B be two (m,n)-upper Q-fuzzy soft subgroups of groups G_1 and G_2 respectively. The Product of A and B denoted by A×B is (m,n)- upper Q-fuzzy soft subgroup of the group $_{G1xG2}$,where (A×B)(x,y)q = max {A(x,q), A(y,q)}, for all (x,y) ε $G_1 \times G_2$.

Proof: Let (x^{-1},a^{-1}) be the inverse element of (x,q) in $G_1 \times G_2$.Then x^{-1} is the inverse element of x in G_1 and a^{-1} is the inverse element of a in G_2.Hence $A(x^{-1},q) \wedge m \leq A(x,q) \vee .n$ and $B(a^{-1},q) \wedge m \leq B(a,q) \vee .n$.

For all (y,b) ε $G_1 \times G_2$.we have

$((A\times B)((x,a)^{-1}(y,b)) \wedge m = ((A\times B)((x^{-1},a^{-1})(y,b))q) \wedge m$

$= \max \{A(x^{-1}y,q), B(a^{-1}b,q)\} \wedge m$

$= \max\{ A(x^{-1}y,q) \wedge m, B(a^{-1}b,q) \wedge m\}$

$\leq \max \{\max (A(x,q), A(y,q)\} \vee .n, \max (B(a,q), B(b,q)\} \vee .n\}$

$= \max \{\max (A(x,q), B(a,q)\}, \max (A(y,q), B(b,q)\}\} \vee .n$

$= \max \{ A\times B (x,a)q , A\times B (y,a)q\}) \vee .n$

Hence A×B is (m,n)-upper Q-fuzzy soft subgroup of $G_1 \times G_2$.

Theorem-3.6: Let A and B be two (m,n)-upper Q-fuzzy soft subsets of groups G_1 and G_2, respectively. If A×B is (m,n)- upper Q-fuzzy soft subgroup of the group $G_1 \times G_2$,then at least one of the following statements must hold.

(i). $A(I_1,q) \wedge m \leq B(a,q) \vee n$, for all a ε G_2 and

(ii). $B(I_2,q) \wedge m \leq A(x,q) \vee n$, for all x ε G_1.

Proof: Let A×B is (m,n)- upper Q-fuzzy soft subgroup of the group $G_1 \times G_2$.

By composition, suppose that none of the statements hold. Then we can find x εG_1 and a ε G_2 such that $A(x,q) \vee n \leq B(I_2,q) \wedge m$ and $B(a,q) \vee n \leq A(I_1,q) \wedge m$. Now

$((A\times B)(x,a))q \vee .n = \max \{ A (x,q) , B (y,q\}\} \vee .n$

$= \max \{ A (x,q) \vee . n , B (y,q) \vee .n\}$

$\leq \max \{ A (I_1,q) \wedge m , B (I_2,q) \wedge m\} = (A\times B)(I_1I_2)q \wedge m.$

Thus A×B is (m,n)- upper Q-fuzzy soft subgroup of the group $G_1 \times G_2$ satisfying $((A\times B)(x,a))q \vee . n \leq (A\times B)(I_1,I_2) \wedge m$. This is a contradict with that (I_1,I_2) is the identity of $G_1 \times G_2$.

Theorem-3.7: Let A and B be Q-fuzzy subsets of groups G_1 and G_2 respectively, such that $B(I_2,q) \wedge m \leq A(x,q) \vee . n$ for all a εG_1.If A×B is a (m,n)-upper Q-fuzzy soft subgroup of $G_1 \times G_2$,then A is a (m,n)-upper Q-fuzzy soft subgroup of G1.

Proof: From $B(I_2,q) \wedge m \leq A(x,q) \vee .n$ we obtain that $m \leq A(x,q) \vee . n$ or $B(I_2,q) \wedge m \leq A(x,q) \vee . n$ for all a εG_1.

Let x,y ε G_1. Then (x,I_2), (y,I_2) ε $G_1 \times G_2$.

Two cases are possible.

(1) If $m \leq A(x, q) \vee .n$ for all a εG_1, then $A(xy, q) \wedge m \leq m \leq A(x, q) \vee . n \leq \max \{ A(x, q, A(y, q\} \vee . n$ and $A (I_1,q) \cap m \leq m \leq A(x, q) \vee . n$.

(2) If $B(I_2,q) \wedge m \leq A(x, q) \vee n$ for all x ε G_1. Then

$A(xy, q) \wedge m \leq (A(xy, q) \vee B(I_2I_2,q)) \wedge m = (A\times B) (xy,I_2I_2)q) \cap m$

$=(A\times B) ((x,I_2)(y,I_2)q)) \wedge m \leq \max \{(A\times B) ((x,I_2)q, (A\times B) ((y,I_2)q\} \vee n.$

$\leq \max \{(A (x,I_2), B (I_2,q) , (A ((y,I_2), B(I_2,q)\} V n. = \max\{A (x, q) , A (y, q)\} \vee n.$

And $A(I,q) \wedge m \leq (A (I1,q) U B (I_2,q)\} \wedge m = ((A\times B)(I_1,I_2)q) \wedge m$

$\leq ((A\times B)(x,I_2)q) \vee n = \max \{ A (x, q) , B (I_2,q)\} \vee n. = A (x, q) \vee n.$

Hence A is (m, n)-upper Q-fuzzy soft subgroup of G_1.

Analogously, we have

Theorem-3.8: Let A and B be Q-fuzzy subsets of groups G_1 and G_2 respectively, such that $A(I_1,q) \wedge m \leq B(a, q) \vee n$ for all a εG_2.If A×B is (m, n)-upper Q-fuzzy soft subgroup of $G_1 \times G_2$,then B is a (m ,n)-upper Q-fuzzy soft subgroup of G_2.

From the Previous theorem, we have the following corollary.

Corollary 3.9: Let A and B be Q-fuzzy subsets of groups G_1 and G_2 respectively, such that $A(I_1,q) \wedge m \leq B(a, q) \vee n$ for all a εG_2.If A×B is (m, n)-upper Q-fuzzy soft subgroup of $G_1 \times G_2$,then either A is (m, n)-upper Q-fuzzy soft subgroup of G_1 or (m, n)-upper Q-fuzzy soft subgroup of G_2.

Section-4 (m, n)-Upper Q- fuzzy index order with fuzzy soft subgroups

Definition 4.1: Let λ be an (m,n)-upper Q-fuzzy soft subgroup of G. For any x ε G, the smallest positive integer n such that $\lambda (x^n) \wedge m = \lambda (e) \vee n$ is called (m,n)- upper Q-fuzzy soft order of x. If there does not exist such n then x is said to have an infinite upper Q- fuzzy soft order. We shall denote the upper Q-fuzzysoft order of x by O (λ (x)).

Example: Let G = { e , a , b , ab } be the Klein four group and let A = { (e , 1/4) , (a , 3 /4) , (b , 3/4) , (ab , 1/4) } be an (m,n)- upper Q-fuzzy soft subgroup, then O (A (ab,q)) = 1 and O (A (a,q)) = 2.

Definition4.2: A (m,n)- upper Q-fuzzy soft subgroup A_H of G is called (m,n)- upper Q- fuzzy soft normal subgroup of G if $A_H(xy,q) \wedge m = A_H(yx,q) \vee n$ for all x,y of G and q ε Q.

Definition4.3: Let A be an (m,n)- upper Q-fuzzy soft subgroup of a group G.A Q-fuzzy soft subset satisfying: $[gA] (x,q) \wedge m = A (g^{-1}x,q) \vee n$ for all x, g ε G; is called an upper Q-fuzzy soft left cosets of A, while that satisfies: $[Ag] (x,q)) \wedge m = A(xg^{-1},q) \vee n$ for all x, g ε G; is called an upper Q-fuzzy soft right cosets of A.

In the following study, the upper Q-fuzzy soft left and right cosets must be fuzzy soft sets but not necessarily be (m,n)- upper Q-fuzzy soft subgroups.

The following propositions are obtained from the above definitions

Proposition 4.1: Let A be an (m,n)- upper Q-fuzzy soft subgroup of the group G. Then for any integer n and x ε G, we have $A(x^n .q)) \wedge m \le A(x,q) \vee n$

Proposition 4.2: Let G be a group and let A be an (m,n)- upper Q-fuzzy soft subgroup of the group G; let x ε G be of finite order k ; if r ε N and k are relatively prime, then $A(x^r,q)) \wedge m = A(x,q) \vee n$.

Proof

Since r, k is relatively prime, then by Bizout Theorem There exists a, b ε Z such that 1 = ar + bk ; therefore $A(x,q)) \wedge m = A(x^{ar+bk},q)) \wedge m = A((x^{ar}.x^{bk}),q)) \wedge m = A(x^{ar},q)) \wedge m \le A(x^r,q) \vee n \le A(x,q) V n$. Then we get $A(x,q)) \wedge m \le A(x r,q)) \wedge m \le A(x,q) \vee n$ therefore $A(xr,q)) \wedge m = A(x,q) \vee n$.

Lemma4.3: Let A be an (m,n)- upper Q- fuzzy soft subgroup of the group G, for x ε G. If $A(e,q)) \wedge m = A(x^n,q) \vee n$ for some n ε Z, then O (A (x, q)) divides n .

Proof

Suppose that O (A (x,q)) = k , then by Euclidean Algorithm , there exists a ,b ε Z . Such that n = ka + b; $0 \le b < k$; thus $A(x^b,q)) \wedge m = A(x^{n-ka},q)) \wedge m = A(x^n(x^k)^{-a},q)) \wedge m \le \max\{A(x^n,q), A(x^k)^{-a},q\} \vee n \le \max\{A(e,q), A(x^k,q)\} \vee n = \max\{A(e,q), A(e,q)\} \vee n = A(e,q) \vee n$. Thus $A(x^b,q)) \wedge m \le A(e,q) \vee n$; also $A(x^b,q)) \wedge m \ge A(e,q) \vee n$, then we get $A(x^b,q)) \wedge m = A(e,q) \vee n$ .Then b = 0; also n =ka which is given O (A (x, q)) divides n.

Proposition 4.4: Let A be an(m,n)- upper Q-fuzzy soft subgroup of the group G, let O (A (x, q)) = A ; x ε G. If k ε Z such m, n relatively prime, then $A(x^k,q) = A(x,q)$.

Proof

Since m, k relatively prime, (m, k) = 1Then there exists a, b ε Z such that ma + kb = 1 $A(x,q)) \wedge m = A(x^{ma+kb},q) = A((x^m)^a . (x^k)^b,q)) \wedge m$

$\le \max\{A(x^m,q)^a, A(x^k,q)^b\} \vee n$

$\le \max\{A(x^m,q), A(x^k,q)\} \vee n$

$\le \max\{A(e,q), A(x^k,q)\}) \vee n$

$= A(x^k,q) \vee n$

$\le A(x,q) \vee n$

Therefore $A(x^k,q)) \wedge m = A(x,q) \vee n$.

Lemma 4.5 : Let A be an(m,n)- upper Q- normal fuzzy soft subgroup of the group G. Then $O(A(x,q))) \wedge m = O(A(y^{-1}xy,q)) \vee n$ for all x , y ε G.

Proof

Let x , y ε G, then we have $A(x^m)) \wedge m = A(y^{-1}x^m y)) \wedge m = A((y^{-1}xy)^m)) \wedge m$, for all m ε Z . Thus $O(A(x,q))) \wedge m = O(A(y^{-1}xy),q) \vee n$.

Remark 4.6: If A is not (m,n)- upper normal Q- fuzzy soft subgroup of the group G, then above lemma 4.5 is not true.

Example: Let G be the Dihedral group

G = { e , a , b , a 2 , ab , ba } and let A = { (e , 1/ 5) , (a , 4 / 5) , (b , 1 / 2) , (a 2 , 4/ 5), (ab , 4 / 5) , (ba , 4 / 5) }Since $O(A(b)) = 1 \ne 2 = O(A(a^{-1}ba))$.

Proposition 4.7: Let G be a finite group and let A, λ be an(m,n)- upper Q-fuzzy soft subgroups, if λ Í m and $A(e,q)) \wedge m = \lambda(e,q) \vee n$ .Then $O(A(x,q))) \wedge m / O(\lambda(x,q)) V n$ for all x ε G such that O (λ (x ,q)) is finite.

Proof

Suppose that O (λ (x,q)) = k , then $A(e,q)) \wedge m = \lambda(e,q)) \wedge m = \lambda(x^n,q)) \wedge m \le A(x^n,q) \vee n$, since $A(e,q)) \wedge m \le A(x^n,q) \vee n$ and $A(e,q)) \wedge m = A(x^n,q) \vee n$. Thus, O (A (x ,q)) / n (by lemma 4.3).

Lemma 4.8: If A is an (m,n)- upper Q-fuzzy soft subgroup of G, then $A(e,q) \wedge m \le A(x,q) \vee n$ for every x ε G.

Proof: $A(e,q)) \wedge m = A((x . x^{-1}),q)) \wedge m \le \max\{A(x,q), A(x^{-1},q)\} \vee n = \max\{A(x,q), A(x,q)\}) \vee n = A(x,q) \vee n$.

Proposition 4.9: Let G be a group and A be an (m,n)- upper Q-fuzzy soft subgroup of G. Then for any x, y ε G such that $A(x,q)) \wedge m \ne A(y,q) \vee n$, we have $A(xy,q)) \wedge m = \max\{A(x,q), A(y,q)\} \vee n$

Proof

Assume that $A(y,q)) \wedge m > A(x,q) \vee n$, then :$A(y,q)) \wedge m = A((x^{-1}xy),q)) \wedge m \le \max\{A(x^{-1},q), A(xy,q)\} \vee n = \max\{A(x,q), A(xy,q)\} \vee n = A(xy,q) \vee n$

Also A is an (m,n)- upper Q-fuzzy soft subgroup, then :

$A(xy,q)) \wedge m \le \max\{A(x,q), A(y,q)\} \vee n = A(y,q) \vee n$. Thus $A(y,q)) \wedge m \le A(xy,q) \vee n \le A(y,q) \vee n$ this implies that $A(xy,q)) \wedge m = \max\{A(x,q), A(y,q)\} \vee n$ The same way if $A(x,q)) \wedge m > A(y,q) \vee n$.

Definition 4.4: A (m,n)- upper Q-fuzzy soft subgroup A of a group G is called (m,n)- upper normal Q-fuzzy soft subgroup if $A((xyx^{-1}),q)) \wedge m \le A(y,q) \vee n$, for all x , y ε G.

Proposition 4.10 The following conditions are equivalent:

(I) G is (m,n)- upper normal Q-fuzzy soft subgroup

(II) $A(xyx^{-1},q)) \wedge m = A(y,q) \vee n$ for all x,y ε G

(III) $A(xy,q)) \wedge m = A(yx,q) \vee n$ for all x,y ε G

Proof

(I) → (II) :

$A(y,q)) \wedge m \ge A(xyx^{-1},q) \vee n$ for all x , y ε G, on the other hand we need to show $A(xyx^{-1},q) \wedge m) \ge A(y,q) \vee n$ for all x ,y ε G. $A(y,q)) \wedge m = A(x^{-1}xyx^{-1}x,q)) \wedge m \le A(xyx^{-1},q) \vee n$. Then $A(y,q)) \wedge m \le A(xyx^{-1},q) \vee n$, therefore $A(xyx^{-1},q)) \wedge m = A(y,q) \vee n$for all x ,y ε G.

(II) → (III) :

Since $xy = x(yx)x^{-1}$, then $A(xy,q)) \wedge m = A(x(yx)x^{-1},q)) \wedge m = A(yx,q) \vee n$, then $A(xy,q)) \wedge m = A(yx,q) \vee n$ for all x ,y ε G.

(III) → (I):

Since $A(xyx^{-1},q)) \wedge m = A((xy)x^{-1},q)) \wedge m = A(x^{-1}(xy),q)) \wedge m = A(x^{-1}xy,q)) \wedge m = A(y,q)) \vee n$, then $A(xyx^{-1},q)) \wedge m \le A(y,q) \vee n$, for all x ,y ε G

Thus the conditions are equivalent.

Definition 4.5 : Let A be an (m,n)- upper Q-fuzzy soft subgroup of a group G. Then the upper Q-fuzzy soft index of A in G is defined by:

The set of the upper Q-fuzzy soft left cosets of A in G, that are themselves are upper Q-fuzzy soft subgroups the set of the upper fuzzy soft left cosets of μ in G,[G:A] = Card

We shall call this type of the upper Q-fuzzy soft index by general upper Q-fuzzy soft index, because this type is not restricted by any conditions.

Proposition4.11: If A is an(m,n)- upper Q-fuzzy soft subgroup of G , then the value of the upper Q-fuzzy soft index is equal to the number of elements in A which is equal to A (e,q) adding for it one , i.e.,

[G : A] = card (element of A which is equal to A (e,q)) + 1).

Proof

For all elements of A which is the value of it's elements equal to A(e,q).The upper Q-fuzzy left cosset of its elements are (m,n)- upper Q-fuzzy soft subgroups; also the upper Q-fuzzy left cosset of A (e,q) is also(m,n)- upper Q-fuzzy soft subgroup. And these satisfy the definition of the upper Q-fuzzy index. Therefore:

[G : A] = card (element of A which is equal to A (e,q) + 1).

Proposition4.12: If A is (m,n)- upper Q-fuzzy soft subgroup of a group G, and [G : A] = 3, then A is an (m,n)-upper normal Q-fuzzy soft subgroup of G.

Proof

Suppose that A is an(m,n)- upper Q-fuzzy soft subgroup of G and [G : A] = 3,this means that A contains two elements having the same value of A(e,q) and let these two elements A (x,q), A (y,q) such that x, y ε G, we need to prove that A is an(m,n)- upper normal Q-fuzzy soft subgroup, which means that

for all z ,h ε G, we need to show A (z h z^{-1},q)) $\wedge$ m $\leq$ A (h,q) $\vee$ n or A (h z,q)) $\wedge$ m = A (zh,q) $\vee$ n or A (h z h^{-1} ,q)) $\wedge$ m $\leq$ A (z ,q) $\vee$ n there exist many cases for A (h,q), A (z,q):

(i) If A (h,q) = A (z,q) and any of these is not equal to A (e,q), then

A (z h z^{-1} ,q)) $\wedge$ m $\leq$ max {A (z h,q), A (z^{-1},q)} $\vee$ n

= max {A (z h,q), A (z,q)} $\vee$ n

$\leq$ max {max {A (z,q), A (h,q)}, A (z,q)} $\vee$ n

= max{max {A (h,q), A (h,q)}, A (h,q)} $\vee$ n

= A (h,q) $\vee$ n

Then A (z h z^{-1},q)) $\wedge$ m $\leq$ A (h,q) $\vee$ n.

(ii) If A (h,q) $\neq$ A (z,q) and any of these is not equal to A (e,q)

A (h z,q)) $\wedge$ m = max {A (h,q), A (z,q)} $\vee$ n and

A (z h,q)) $\wedge$ m = max{A (z,q), A (h,q)} $\vee$ n thus

A (z h,q)) $\wedge$ mn = max {A (z,q), A (h,q)} $\vee$ n = max {A (h,q), A (z,q)} $\vee$ n =A (h z,q) $\vee$ n, then A (z h,q)) $\wedge$ m = A (h z,q) $\vee$ n.

(iii) In this case we have two sub cases :

Sub case (a). In the case if the element h of G is the same of x or y or e it follows that A (h,q) = A (e,q) and the element z of G is different from x, y and e then A (e,q) $\lneqq$ A (z,q) because there are only two elements in G and their values in A equal to A (e,q), thus A (h z h^{-1},q)) $\wedge$ m

$\leq$ max {A (h z,q),A(h^{-1},q)} $\vee$ n= max {A (h z,q), A (h,q)} $\vee$ n

$\leq$ max {max {A (h,q), A (z,q), A (h,q)} $\vee$ n

= max {A (z,q), A (h,q)} $\vee$ n

= A (z,q) $\vee$ n Then A (h z h^{-1},q)) $\wedge$ m $\leq$ A (z,q) $\vee$ n .

Subcase (b). In the case if the element z of G is the same of x or y or e it follows that A (z,q) = A (e) and the element h of G is different from x, y and e then A (e,q) $\lneqq$ A(h,q) because there are only two elements in G and their values in μ equal to μ (e), thus A (z h z^{-1} ,q)) $\wedge$ m $\leq$ max{A (z h,q),A (z^{-1},q)} $\vee$ n= max {A (z h,q), A (z,q)} $\vee$ n $\leq$ max {max {A (z,q), A (h,q), A (z,q)} $\vee$ n

= max{A (h,q), A (z,q)}= A (h,q) $\vee$ n

Then A (z h z^{-1},q)) $\wedge$ m $\leq$ A (h,q) $\vee$ n.

(iv) if the element h of G is the same of x or y or e it follows that A (h,q) = A (e,q), and the element z of G is the same of x or y or e it follows that A (z,q) = A (e,q) then A (z h z^{-1} ,q)) $\wedge$ m $\leq$ max{A(z h,q),A(z^{-1},q)} $\vee$ n= max {A (z h,q), A (z ,q)} $\vee$ n$\leq$ max {max {A (z,q), A (h,q)}, A (z,q)} $\vee$ n

= max{max {A (h,q), A (h,q)}, A (h,q)} $\vee$ n= A (h,q) $\vee$ n

Then A (z h z^{-1},q)) $\wedge$ m $\leq$ A (h,q) $\vee$ n .

From the above, all cases that we have studied, we conclude that A is (m,n)- upper normal Q-fuzzy soft subgroup of G.

4. Conclusion

We have studied in this paper the definition of the (m,n)- upper Q-fuzzy subgroup over an arbitrary group. Some proposition, theorems and examples given for it and this proposition and corollary are generalization for some properties in group theory.

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Biography

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